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Analysis of Grasp Parameter Effects for Static Stability of Planar Grasps

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Abstract—Grasp stability is an important factor for grasping and manipulation. In this paper, we discuss error analysis of grasp parameters for static grasp stability in two dimensions. Each finger is modeled by translational linear springs, and the potential energy stored in the grasp is formulated. The force and moment vector and the stiffness matrix of the grasp are derived by using the first- and second-order partial derivatives of the energy. The vector and the matrix are represented by a function of grasp parameters such as contact condition (rolling contact and sliding contact), contact point, local curvature, finger spring stiffness, and so on. The partial derivatives of the vector and the matrix with respect to the grasp parameters are derived. The characteristics of the parameters' effects are investigated by using positive definiteness of the derivatives. It is analytically shown that the stability is enhanced when the rolling contact appears, the local curvature decreases, the finger stiffness increases, and so on. The directions affecting the stability by the parameters' deviation are also derived. The effects of the other parameters are also shown.

Keywords—Grasp stability, grasp stiffness matrix, grasp parameters, error analysis.

I. Introduction

HUMAN hands grasp and manipulate various shape of objects dexterously. Multi-fingered robot hands have similar potential. When grasped objects are displaced by an external disturbance, the hands have to keep the grasp stable. Grasp stability is an important factor for grasping and manipulation.

Hanafusa and Asada [1] formulated an object grasped by elastic mechanical fingers. Each finger was replaced by linear spring model, and potential energy stored in the grasp is provided. They pointed out that the grasp is stable if the potential energy is local minimum. From this viewpoint, the first- and second-order partial derivatives of the energy are derived. The first-order partial derivative provides force and moment vectors, the second-order partial derivative provide a stiffness matrix of the grasp. Nguyen [2] pointed out that the spring model can be generated using computer control of the finger position. Not only two dimensional but also three dimensional grasps are formulated. Montana [3] pointed out that local curvature affects grasp stability. Howard and Kumar [4] formulated three dimensional grasp including local curvature in the potential energy method. Rimon and Burdick [5] also consider the local curvature in the analysis of immobility of the bodies. In their works, each rolling contact finger is modeled by multiple springs and each sliding contact finger is modeled by a single spring along the contact normal direction.

Yamada et al. [6] pointed out that the difference between rolling and sliding contacts is not given by spring model but given by contact point motion. From this viewpoint, the multiple springs' model was applied not only to the rolling contact but also to the sliding contact. Yamada et al. [7] included contact surface geometry (metric tensor, curvature, torsion) in three dimensional grasps. Moreover, they extended to grasping of multiple objects [8], [9] and consideration of finger links [10]. In their works, the grasp stiffness matrix was analytically derived. The matrix is represented by a function of grasp parameters such as contact condition (rolling contact and sliding contact), contact point, contact force, local curvature, finger spring stiffness, and so on.

In this paper, we analyze the effects of the grasp parameters on the grasp stability. For the first time, this analysis was presented in Reference [11]. In the analysis, the orientation of the finger stiffness is adjusted to the contact normal and tangential directions. This paper treats more general case that the orientation is tilted from the directions. And further analyses are provided. In Section II, we explain the coordinated frames used in this paper. The relation of the object poses displacement and the finger position displacement. The potential energy stored in the grasp is formulated. The force and moment vector and the stiffness matrix of the grasp are provided by using the first- and second-order partial derivatives of the energy. In Section III, the effects of the grasp parameters are analyzed. The partial derivatives of the vector and the matrix are derived. Positive definiteness of the derivatives is investigated. In Section IV, we conclude this paper.

II. Fundamental Formulation

We consider the grasp shown in **Figure 1**. In this section, we briefly explain the derivation of the stiffness matrix of the grasp.

A. Assumptions

Based on the following assumptions, we formulate the grasp system.

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(A1) The object and the fingers are rigid bodies.
(A2) A single point contact exists on each finger.
(A3) The contact location, orientation, and local curvature are given.
(A4) An infinitesimal pose displacement of the object occurs due to an external disturbance.
(A5) Each finger is modeled as a translational linear spring model.

B. Nomenclature

The symbol $\Sigma_{\bullet}$ represents a coordinate frame. Σ_b is a base frame fixed in the space. Σ_o is an object frame fixed in the object. Σ_{fk} is a finger frame fixed in the *k*-th finger. Σ_{Cfk} is a current contact frame moving on the finger surface. Σ_{Cok} is a current contact frame moving on the object surface. Frames Σ_{bo}, Σ_{bfk}, Σ_{Lfk} and Σ_{Lok} are given at the initial poses of Σ_o, Σ_{fk}, Σ_{Cfk} and Σ_{Cok}, respectively.

Homogeneous transformation matrices used in this paper are described in Appendix A. Other vectors and matrices are also described in the appendix.

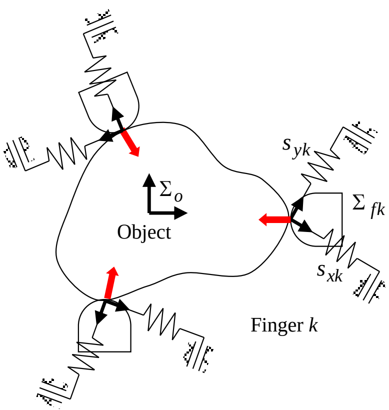


Figure 1 An object grasped by multi-fingers with translational linear spring model

C. Pose displacement of frames

The pose displacement relation between the fingers and the object is formulated with the matrices.

C.1. Local contact frame

The local contact frame on the object surface, Σ_{Lok}, is represented by

$$ {}^{b}A_{Lok}(\boldsymbol{\varepsilon}_o) = {}^{b}A_{bo}\,{}^{bo}A_{o}(\boldsymbol{\varepsilon}_o)\,{}^{o}A_{Lok} \tag{1}$$

where $\boldsymbol{\varepsilon}_o = [\boldsymbol{x}_o^T, \zeta_o]^T$ is the object's pose displacement. Hence, we have ${}^{bo}A_o(\boldsymbol{\varepsilon}_o) = A_{trans}(\boldsymbol{x}_o)A_{rot}(\zeta_o)$. The local contact frame on the finger is calculated by

$$ {}^{b}A_{Lfk}(\boldsymbol{\varepsilon}_{fk}) = {}^{b}A_{bfk}\,{}^{bfk}A_{fk}(\boldsymbol{\varepsilon}_{fk})\,{}^{fk}A_{Lfk} \tag{2}$$

where ${}^{bfk}A_{fk}(\boldsymbol{\varepsilon}_{fk})$ is defined in a similar manner of ${}^{bo}A_o(\boldsymbol{\varepsilon}_o)$

C.2. Current contact frame

From (A3), the current contact frame on the object surface with respect to the local frame is defined by

$$ {}^{Lok}A_{Cok}(\alpha_{ok}) = {}^{Lok}A_{\kappa ok}A_{rot}(\kappa_{ok}\alpha_{ok})\,{}^{Lok}A_{\kappa ok}^{-1} \tag{3}$$

where ${}^{Lok}A_{\kappa ok} := A_{trans}(-\kappa_{ok}^{-1}\boldsymbol{u}_1)$. The symbol κ represents the local curvature, and α is the contact location displacement on the body (**Figure 2**). The convex, flat, and concave surfaces are represented by $\kappa > 0$, $\kappa = 0$, and $\kappa < 0$, respectively. The current contact frame on the finger surface, ${}^{Lfk}A_{Cfk}(\alpha_{fk})$, is defined in a similar manner of (3).

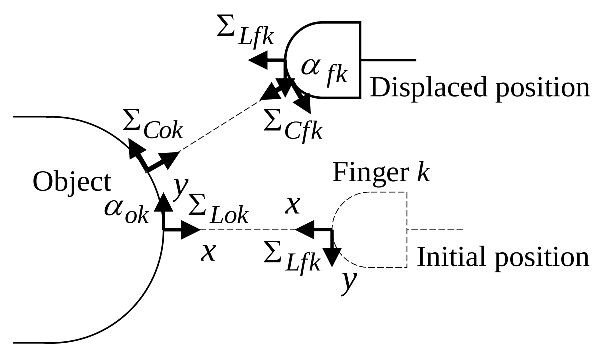


Figure 2 The local and current contact frames on the object and the finger

C.3. Contact constraint

By using these homogeneous matrices, the contact constraint between the finger and the object is given by

$$ {}^{b}A_{Lfk}(\boldsymbol{\varepsilon}_{fk})\,{}^{Lfk}A_{Cfk}(\alpha_{fk})\,{}^{Cfk}A_{Cok} = {}^{b}A_{Lok}(\boldsymbol{\varepsilon}_o)\,{}^{Lok}A_{Cok}(\alpha_{ok}) \tag{4}$$

where $\kappa_{fk} + \kappa_{ok} > 0$ and ${}^{Cfk}A_{Cok} = A_{rot}(\pi)$ from (A2). From (4), the finger pose displacement is obtained by

$$ {}^{bfk}A_{fk}(\boldsymbol{\varepsilon}_{fk}) = {}^{bfk}A_{bo}\,{}^{bo}A_o(\boldsymbol{\varepsilon}_o)\,{}^{o}A_{Lok}\,{}^{Lok}A_{Cok}(\alpha_{ok}) \times {}^{Cok}A_{Cfk}\,{}^{Lfk}A_{Cfk}^{-1}(\alpha_{fk})\,{}^{Lfk}A_{fk} \tag{5}$$

In (5), $\boldsymbol{\varepsilon}_{fk}$ is given by a function of $\boldsymbol{\varepsilon}_o$, α_{ok} and α_{fk}. Because the finger pose displacement is modeled as the spring, the parameters α_{ok} and α_{fk} depend on $\boldsymbol{\varepsilon}_o$ and then are redundant.

From Assumption (A5), the finger orientation is constrained by

$$ {}^{bfk}R_{fk} = I_{23}\,{}^{bfk}A_{fk}(\boldsymbol{\varepsilon}_{fk})I_{23}^T = I_2 \tag{6}$$

Because (6) provides one valid constraint, α_{fk} is selected as a redundant parameter and eliminated. After that, the translation displacement of the finger is given by

$$\boldsymbol{x}_{fk}(\boldsymbol{\varepsilon}_o,\alpha_{ok}) := {}^{bfk}\boldsymbol{p}_{fk} = I_{23}\,{}^{bfk}A_{fk}(\boldsymbol{\varepsilon}_{fk})\boldsymbol{v}_{\zeta} \tag{7}$$

D. Grasp stiffness matrix

D.1. Potential energy of the finger

The potential energy of the finger with translational linear spring model is calculated by

$$U_k(\boldsymbol{\varepsilon}_o,\alpha_{ok}) = \tfrac{1}{2}[\boldsymbol{x}_{fk0} + \boldsymbol{x}_{fk}(\boldsymbol{\varepsilon}_o,\alpha_{ok})]^T \times S_k[\boldsymbol{x}_{fk0} + \boldsymbol{x}_{fk}(\boldsymbol{\varepsilon}_o,\alpha_{ok})] \tag{8}$$

where the vector $\boldsymbol{x}_{fk0}$ is a spring compression. The matrix $S_k = \mathrm{diag}[\,s_{xk}, s_{yk}\,]$ is the spring model described in (A5). The finger force vector is given by

$$\boldsymbol{f}_k = [\,f_{xk}, f_{yk}\,]^T := S_k\boldsymbol{x}_{fk0} \tag{9}$$

This force implies a reaction force from the object. These vectors and matrix are defined in the frame Σ_{bfk}.

The rolling and sliding contacts between the finger and object are included, then we have

$$U_k^{fc}(\boldsymbol{\varepsilon}_o) := U_k(\boldsymbol{\varepsilon}_o, \alpha_{ok}^{fc}(\boldsymbol{\varepsilon}_o)) \tag{10}$$

The total potential energy of the grasp with *n* fingers is given by

$$U(\boldsymbol{\varepsilon}_o) = U_e(\boldsymbol{\varepsilon}_o) + \sum_{k=1}^{n} U_k^{fc}(\boldsymbol{\varepsilon}_o) \tag{11}$$

where $U_e(\boldsymbol{\varepsilon}_o)$ is the potential energy affected by an external force. The gradient and the hessian of the energy are given by

$$G = G_e + \sum_{k=1}^{n} G_k^{fc}, \quad H = H_e + \sum_{k=1}^{n} H_k^{fc} \tag{12}$$

where

$$G := \left.\frac{\partial U(\boldsymbol{\varepsilon}_o)}{\partial \boldsymbol{\varepsilon}_o}\right|_0, \quad H := \left.\frac{\partial^2 U(\boldsymbol{\varepsilon}_o)}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\varepsilon}_o^T}\right|_0 \tag{13}$$

The superscript "*fc*" represents "*fr*" or "*fs*" described in Sections D.2 and D.3.

The gradient means the force and moment vector, and the hessian is called the stiffness matrix of the grasp. The grasp is stable if *G* = 0 and *H* is positive definite. The grasp stability is evaluated by the eigenvalues of the matrix *H*, and the grasp displacement direction is given by the corresponding eigenvectors.

D.2. Rolling contact case

In the case of rolling contact, the finger rolls on the object's surface, then we have

$$\alpha_{ok} + \alpha_{fk} = 0 \tag{14}$$

The first-order partial derivative of (10) is given by

$$G_k^{fr} = {}^{o}W_{Lfk}\,{}^{Lfk}\boldsymbol{f}_k \tag{15}$$

where ${}^{Lfk}\boldsymbol{f}_k := {}^{Lfk}R_{bfk}\boldsymbol{f}_k$. Because the finger can not pull the object, we have

$${}^{Lfk}f_{xk} := {}^{Lfk}\boldsymbol{f}_k^T\boldsymbol{u}_1 < 0 \tag{16}$$

Inequality (16) is given by the reason why the x axis of the frame Σ_{Lfk} directs along the outward normal direction of the finger surface but the reaction force directs opposite to the direction. The normal component magnitude of the contact force is obtained by $-{}^{Lfk}f_{xk}$.

The second-order partial derivative of (10) is given by

$$H_k^{fr} = {}^{o}W_{Lfk}\,{}^{Lfk}S_k\,{}^{o}W_{Lfk}^T + {}^{Lfk}\boldsymbol{f}_k^T\left[{}^{Lfk}\boldsymbol{p}_o - \frac{\boldsymbol{u}_1}{\kappa_{ok}+\kappa_{fk}}\right]\boldsymbol{v}_{\zeta}\boldsymbol{v}_{\zeta}^T \tag{17}$$

where ${}^{Lfk}S_k := {}^{Lfk}R_{bfk}S_k\,{}^{bfk}S_{Lfk}$.

D.3. Sliding contact case

In the sliding contact case, the finger slides on the object surface to locally minimize the potential energy. Then we have

$$\frac{\partial U_k(\boldsymbol{\varepsilon}_o,\alpha_{ok})}{\partial \alpha_{ok}} = 0, \quad \frac{\partial^2 U_k(\boldsymbol{\varepsilon}_o,\alpha_{ok})}{\partial \alpha_{ok}\partial \alpha_{ok}} > 0 \tag{18}$$

From the initial condition of the first equation of (18), the tangential component of the contact force is zero.

$${}^{Lfk}f_{yk} := {}^{Lfk}\boldsymbol{f}_k^T\boldsymbol{u}_2 = 0 \tag{19}$$

From the initial condition of the inequality of (18), the following condition has to be satisfied to stabilize the finger with sliding contact.

$${}^{Lfk}s'_{k22} := {}^{Lfk}s_{k22} + \frac{\kappa_{ok}\kappa_{fk}}{\kappa_{ok}+\kappa_{fk}}\,{}^{Lfk}f_{xk} > 0 \tag{20}$$

where ${}^{Lfk}s_{k22} := \boldsymbol{u}_2^T\,{}^{Lfk}S_k\boldsymbol{u}_2$.

The first-order partial derivative of (10) is given by

$$G_k^{fs} = {}^{o}W_{Lfk}\,{}^{Lfk}\boldsymbol{f}_k = {}^{o}W_{Lfk}\boldsymbol{u}_1\,{}^{Lfk}f_{xk} \tag{21}$$

The second-order partial derivative is given by

$$\begin{aligned} H_k^{fs} &= {}^oW_{Lfk}\, {}^{Lfk}S_k\, {}^oW_{Lfk}^T \\ &+ {}^{Lfk}f_{xk}\left\{ {}^{Lfk}\boldsymbol{p}_o^T \boldsymbol{u}_1 - \frac{1}{\kappa_{ok}+\kappa_{fk}} \right\} \boldsymbol{v}_\zeta \boldsymbol{v}_\zeta^T \\ &- \frac{1}{{}^{Lfk}s'_{k22}} \left[{}^oW_{Lfk}\, {}^{Lfk}S_k \boldsymbol{u}_2 - \frac{\kappa_{fk}\, {}^{Lfk}f_{xk}}{\kappa_{ok}+\kappa_{fk}} \boldsymbol{v}_\zeta \right] \\ &\times \left[{}^oW_{Lfk}\, {}^{Lfk}S_k \boldsymbol{u}_2 - \frac{\kappa_{fk}\, {}^{Lfk}f_{xk}}{\kappa_{ok}+\kappa_{fk}} \boldsymbol{v}_\zeta \right]^T \end{aligned} \tag{22}$$

III. Effects of Parameter Deviation

A. Effect of contact condition

To compare the rolling contact case and the sliding one, we consider the same condition between the rolling and the sliding. Hence, we let ${}^{Lfk}f_{yk} = 0$.

The effect of contact condition is given by

$$G_k^{fd} := G_k^{fr} - G_k^{fs} = 0_{3\times1} \tag{23}$$

and

$$H_k^{fd} := H_k^{fr} - H_k^{fs} = \frac{1}{{}^{Lfk}s'_{k22}} X_{Lfk}^{fd} [X_{Lfk}^{fd}]^T \tag{24}$$

Because the matrix H_k^{fd} is positive semi-definite, the stability is enhanced by the rolling contact. The effect appears in the following displacement direction:

$$X_{Lfk}^{fd} := {}^oW_{Lfk}\, {}^{Lfk}S_k \boldsymbol{u}_2 - \frac{\kappa_{fk}\, {}^{Lfk}f_{xk}}{\kappa_{ok}+\kappa_{fk}} \boldsymbol{v}_\zeta \tag{25}$$

This direction depends on the finger stiffness, the normal direction force, the curvatures, and so on. We define the following vectors:

$$\begin{aligned} Y_{Lfk}^{fd} &:= \boldsymbol{v}_\zeta \otimes X_{Lfk}^{fd} = I_{23}^T \Omega\, {}^oR_{Lfk}\, {}^{Lfk}S_k \boldsymbol{u}_2, \\ Z_{Lfk}^{fd} &:= X_{Lfk}^{fd} \otimes Y_{Lfk}^{fd} \end{aligned} \tag{26}$$

These vectors are base vectors of object's displacement in the null space of (24) because the following conditions are satisfied:

$$[Y_{Lfk}^{fd}]^T H_k^{fd} Y_{Lfk}^{fd} = 0, \quad [Z_{Lfk}^{fd}]^T H_k^{fd} Z_{Lfk}^{fd} = 0 \tag{27}$$

This means that the stability in the object's displacement of (26) does not depend on the deviation of friction condition. In this grasp, the total differences are given by

$$\begin{aligned} G^{fd} &= \sum_{k=1}^n G_k^{fd} = 0_{3\times1}, \\ H^{fd} &= \sum_{k=1}^n H_k^{fd} = X_{Lf}^{fd}\, {}^{Lf}S'^{-1}_{22} [X_{Lf}^{fd}]^T \end{aligned} \tag{28}$$

where

$$\begin{aligned} X_{Lf}^{fd} &:= [X_{Lf1}^{fd}, \cdots, X_{Lfn}^{fd}], \\ {}^{Lf}S'_{22} &:= \mathrm{diag}[{}^{Lf1}s'_{122}, \cdots, {}^{Lfn}s'_{n22}] \end{aligned} \tag{29}$$

Example: Two fingered simple grasp

In order to clarify the effects, we consider the simple grasp shown in **Figure 3**. The stiffness orientation is set to ${}^{Lfk}R_{bfk} = -I_2$. All fingers are set to the same values as $S_k = \mathrm{diag}[s_x, s_y]$, $\kappa_{fk} = \kappa_f$ and $\kappa_{ok} = \kappa_o$. The contact locations are given by

$${}^oA_{Lf1} = \begin{bmatrix} -I_2 & l_x \boldsymbol{u}_1 \\ 0_{1\times2} & 1 \end{bmatrix}, \quad {}^oA_{Lf2} = \begin{bmatrix} I_2 & -l_x \boldsymbol{u}_1 \\ 0_{1\times2} & 1 \end{bmatrix} \tag{30}$$

From (29), we have

$$X_{Lf}^{fd} = \begin{bmatrix} -s_y \boldsymbol{u}_2 & s_y \boldsymbol{u}_2 \\ \dfrac{\kappa_f f_x}{\kappa_o + \kappa_f} - s_y l_x & \dfrac{\kappa_f f_x}{\kappa_o + \kappa_f} - s_y l_x \end{bmatrix} \tag{31}$$

The base vector of the object displacement in the null space of H^{fd} is given by $\boldsymbol{v}_x$. This displacement direction exists in the normal direction at the contact points (**Figure 4(a)**). In the other directions shown in **Figure 4(b)** and **Figure 4(c)**, the grasp is affected by the deviation of contact friction condition.

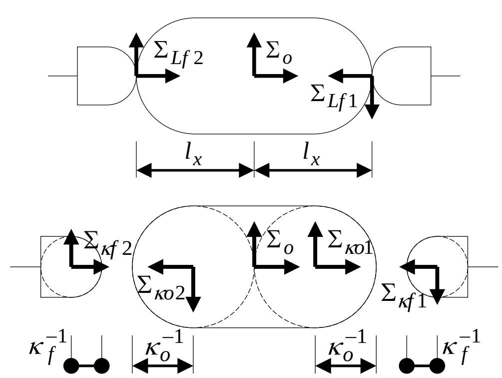
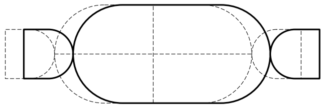
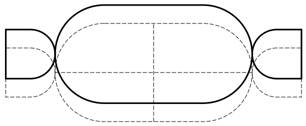


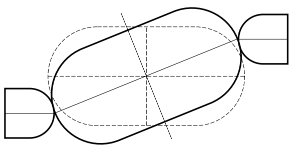
Figure 3 An example of a two-fingered grasp



(a) Displacement of $\boldsymbol{v}_x$



(b) Displacement of $\boldsymbol{v}_y$



(c) Displacement of $\boldsymbol{v}_\zeta$

Figure 4 The object's displacements

B. Local curvature effect

We investigate effects of the object's local curvature κ_{ok} and the finger's local curvature κ_{fk}.

B.1 Rolling contact

In the rolling contact case, the first-order partial derivatives of (15) with respect to the local curvatures are given by

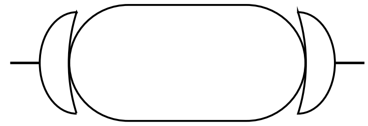
$$\frac{\partial G_k^{fr}}{\partial \kappa_{ok}} = 0_{3\times1}, \quad \frac{\partial G_k^{fr}}{\partial \kappa_{fk}} = 0_{3\times1} \tag{32}$$

The force and moment vector does not depend on the deviation of the local curvature. The first-order partial derivatives of (17) are given by

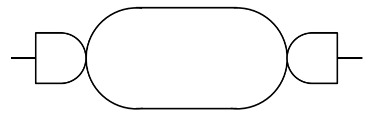
$$\begin{aligned} \frac{\partial H_k^{fr}}{\partial \kappa_{ok}} &= \frac{{}^{Lfk}f_{xk}}{(\kappa_{ok}+\kappa_{fk})^2}\boldsymbol{v}_\zeta \boldsymbol{v}_\zeta^T, \\ \frac{\partial H_k^{fr}}{\partial \kappa_{fk}} &= \frac{{}^{Lfk}f_{xk}}{(\kappa_{ok}+\kappa_{fk})^2}\boldsymbol{v}_\zeta \boldsymbol{v}_\zeta^T \end{aligned} \tag{33}$$

Because these derivatives are negative semi-definite, the grasp stability decreases when the local curvatures increase. This means that the stability of concave shape (**Figure 5(a)** and **Figure 5(b)**) is more stable than that of convex shape (**Figure 5(c)**). The effect appears in the displacement direction of $\boldsymbol{v}_\zeta$. This direction means the rotation at the origin of the object frame Σ_o (**Figure 4(c)**). In this displacement, the contact location moves on the object and finger surfaces. The base vectors of the object displacement in the null space of (33) are given by $\boldsymbol{v}_x$ and $\boldsymbol{v}_y$. In these displacements (**Figure 4(a)** and **Figure 4(b)**), the contact locations do not move on the object and the finger surfaces, the stability is not affected by the curvature deviations.

(a) Concave object – convex finger contact



(b) Convex object – concave finger contact



(c) Convex object – convex finger contact

Figure 5 Difference in the local curvature of the object and finger surfaces

B.2 Sliding contact

In the sliding contact case, the first-order partial derivatives of (21) are given by

$$\frac{\partial G_k^{fs}}{\partial \kappa_{ok}} = 0_{3\times1}, \quad \frac{\partial G_k^{fs}}{\partial \kappa_{fk}} = 0_{3\times1} \tag{34}$$

The force and moment vectors do not depend on the local curvature deviation. The first-order partial derivative of (22) is given by

$$\frac{\partial H_k^{fs}}{\partial \kappa_{ok}} = {}^{Lfk}f_{xk} Q_k^{fs} [Q_k^{fs}]^T \tag{35}$$

where

$$Q_k^{fs} := \left. \frac{\partial \alpha_{ok}^{fs}(\boldsymbol{\varepsilon}_o)}{\partial \boldsymbol{\varepsilon}_o} \right|_0 = \frac{\kappa_{fk} X_{\kappa fk}}{(\kappa_{ok}+\kappa_{fk})^{Lfk} s'_{k22}} \tag{36}$$

Because (35) is negative semi-definite, the grasp stability decreases when the local curvature of the object increases. The effect appears in the displacement direction of Q_k^{fs}. We define the following vector:

$$X_{\kappa fk} := {}^{o}W_{\kappa fk} {}^{\kappa fk}S_k \boldsymbol{u}_2 - {}^{\kappa fk}f_{xk} \boldsymbol{v}_\zeta \tag{37}$$

where

$$\begin{aligned} {}^{\kappa fk}S_k &:= {}^{\kappa fk}R_{bfk} S_k {}^{bfk}S_{\kappa fk} = {}^{Lfk}S_k, \\ {}^{\kappa fk}\boldsymbol{f}_k &= [{}^{\kappa fk}f_{xk}, {}^{\kappa fk}f_{yk}]^T := {}^{\kappa fk}R_{bfk}\boldsymbol{f}_k = {}^{Lfk}\boldsymbol{f}_k \end{aligned} \tag{38}$$

Note that ${}^{\kappa fk}R_{Lfk} = I_2$ because the frame $\Sigma_{\kappa fk}$ is fixed at the curvature center parallel to Σ_{Lfk}. The base vectors of object displacement in the null space of (35) are given by

$$\begin{aligned} Y_{\kappa fk} &:= \boldsymbol{v}_{\zeta} \otimes X_{\kappa fk} = I_{23}^{T}\Omega\, {}^{o}R_{\kappa fk}\, {}^{\kappa fk}S_k \boldsymbol{u}_2, \\ Z_{\kappa fk} &:= X_{\kappa fk} \otimes Y_{\kappa fk} \\ &= \begin{bmatrix} [{}^{\kappa fk}f_{xk} - {}^{\kappa fk}\boldsymbol{p}_o^{T}\Omega\, {}^{\kappa fk}S_k \boldsymbol{u}_2]\, {}^{o}R_{\kappa fk}\, {}^{\kappa fk}S_k \boldsymbol{u}_2 \\ \left\| {}^{\kappa fk}S_k \boldsymbol{u}_2 \right\|^2 \end{bmatrix} \end{aligned} \tag{39}$$

The derivations of (39) are shown in Appendix B. In these directions of the object displacement, the stability are not affected by the object's curvature deviation.

The partial derivative of the stiffness matrix with respect to the finger's local curvature is given by

$$\frac{\partial H_k^{fs}}{\partial \kappa_{fk}} = \frac{{}^{Lfk}f_{xk}\kappa_{ok}^2 X_{\kappa okS2} X_{\kappa okS2}^T}{\{(\kappa_{ok}+\kappa_{fk})\, {}^{Lfk}s'_{k22}\}^2} \tag{40}$$

Because (40) is negative semi-definite, the stability decreases when the local curvature of the finger increases. The effect appears in the following direction:

$$X_{\kappa okS2} := {}^{o}W_{\kappa ok}\, {}^{\kappa ok}S_k \boldsymbol{u}_2 \tag{41}$$

The base vectors of object displacement in the null space of (40) are given by

$$\begin{aligned} Y_{\kappa okS2} &:= \boldsymbol{v}_{\zeta} \otimes X_{\kappa okS2} = I_{23}^{T}\Omega\, {}^{o}R_{\kappa ok}\, {}^{\kappa ok}S_k \boldsymbol{u}_2, \\ Z_{\kappa okS2} &:= X_{\kappa okS2} \otimes Y_{\kappa okS2} \\ &= \begin{bmatrix} -[{}^{o}R_{\kappa ok}\, {}^{\kappa ok}S_k \boldsymbol{u}_2][{}^{\kappa ok}\boldsymbol{p}_o^{T}\Omega\, {}^{\kappa ok}S_k \boldsymbol{u}_2] \\ \left\| {}^{\kappa ok}S_k \boldsymbol{u}_2 \right\|^2 \end{bmatrix} \end{aligned} \tag{42}$$

These directions are not affected by the deviation of finger's local curvature. In the case of ${}^{Lfk}R_{bfk} = -I_2$, these directions are illustrated in Figure 6.

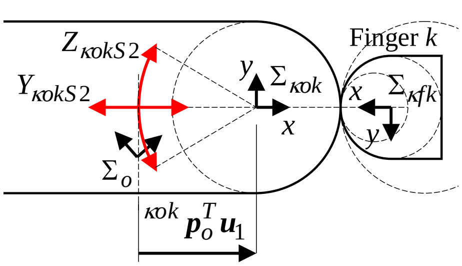


Figure 6 Finger's curvature deviation and object's displacement of (42)

The first-partial derivative of ${}^{Lfk}s'_{k22}$ w.r.t. the local curvatures are given by

$$\frac{\partial\, {}^{Lfk}s'_{k22}}{\partial \kappa_{ok}} = \frac{\kappa_{fk}^2\, {}^{Lfk}f_{xk}}{(\kappa_{ok}+\kappa_{fk})^2}, \quad \frac{\partial\, {}^{Lfk}s'_{k22}}{\partial \kappa_{fk}} = \frac{\kappa_{ok}^2\, {}^{Lfk}f_{xk}}{(\kappa_{ok}+\kappa_{fk})^2} \tag{43}$$

Because these derivatives are not positive, the condition (20) is enhanced when the local curvatures decreases. This means that the value of ${}^{Lfk}s'_{k22}$ of the grasps shown in **Figure 5(a)** and **Figure 5(b)** are larger than that of **Figure 5(c)**.

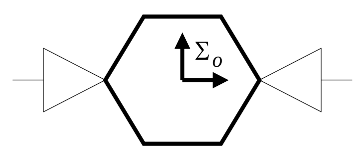
If the local curvatures are positively infinite, we have

$$\lim_{\substack{\kappa_{ok}\to+\infty \\ \kappa_{fk}\to+\infty}} {}^{Lfk}s'_{k22} = -\infty \tag{44}$$

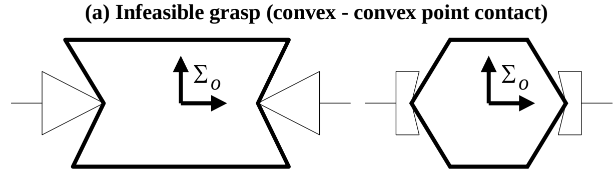
In this case, the condition (20) is not satisfied. This means that the grasp shown in **Figure 7(a)** is infeasible. Robots have to avoid grasping the convex vertices if the convex fingers are used. When the local curvatures are given by both positive and negative infinites, we have

$$\lim_{\substack{\kappa_{ok}\to+\infty \\ \kappa_{fk}\to-\infty}} {}^{Lfk}s'_{k22} = +\infty, \quad \lim_{\substack{\kappa_{ok}\to-\infty \\ \kappa_{fk}\to+\infty}} {}^{Lfk}s'_{k22} = +\infty \tag{45}$$

This means that the grasps shown in **Figure 7(b)** are feasible. After the condition (20) is checked, the grasp stability is evaluated by using (22).



(a) Infeasible grasp (convex - convex point contact)



(b) Feasible grasp (convex – concave point contact)

Figure 7 Evaluation of the grasps by (20)

C. *Stiffness effect of the finger*

The stiffness matrix is represented by

$$S_k = s_{xk}\boldsymbol{u}_1\boldsymbol{u}_1^T + s_{yk}\boldsymbol{u}_2\boldsymbol{u}_2^T \tag{46}$$

We investigate effects of s_{xk} and s_{yk}. In this analysis, we consider that the force $\boldsymbol{f}_k$ is independent of the stiffness S_k because $\boldsymbol{f}_k$ depends not only on S_k but also on $\boldsymbol{x}_{fk0}$.

C.1. Rolling contact

In the rolling contact case, the partial derivative of (15) is given by

$$\frac{\partial G_k^{fr}}{\partial s_{xk}} = 0_{3\times1}, \quad \frac{\partial G_k^{fr}}{\partial s_{yk}} = 0_{3\times1} \tag{47}$$

The force and moment vectors do not depend on the stiffness deviation. The partial derivatives of (17) are given by

$$\frac{\partial H_k^{fr}}{\partial s_{xk}} = X_{Lfk1} X_{Lfk1}^T \tag{48}$$

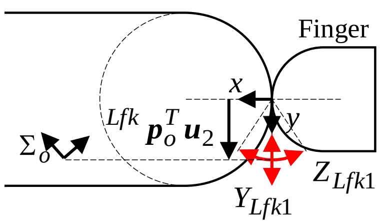
$$\frac{\partial H_k^{fr}}{\partial s_{yk}} = X_{Lfk2} X_{Lfk2}^T \tag{49}$$

Because these derivatives are positive semi-definite, the stability is enhanced when the stiffness is increased. The effects appear in the following directions

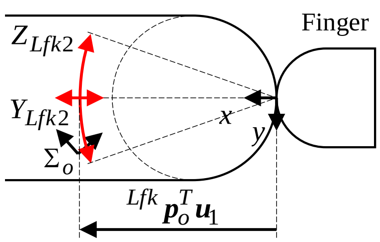
$$X_{Lfk1} := {}^{o}W_{Lfk} \, {}^{Lfk}R_{bfk} \boldsymbol{u}_1, \quad X_{Lfk2} := {}^{o}W_{Lfk} \, {}^{Lfk}R_{bfk} \boldsymbol{u}_2 \tag{50}$$

The base vectors of the object displacement in the null space of (48) are given by

$$\begin{aligned} Y_{Lfk1} &:= \boldsymbol{v}_\zeta \otimes X_{Lfk1} = I_{23}^T \, {}^{o}R_{bfk} \boldsymbol{u}_2, \\ Z_{Lfk1} &:= X_{Lfk1} \otimes Y_{Lfk1} = \begin{bmatrix} -{}^{o}R_{bfk} \boldsymbol{u}_1 [{}^{Lfk}\boldsymbol{p}_o^T \, {}^{Lfk}R_{bfk} \boldsymbol{u}_2] \\ 1 \end{bmatrix} \end{aligned} \tag{51}$$



(a) Object displacement of Y_{Lfk1} and Z_{Lfk1}



(b) Object displacement of Y_{Lfk2} and Z_{Lfk2}

Figure 8: Displacement direction of (51) and (52)

In these displacements, the length of spring s_{xk} does not change. In the case of ${}^{Lfk}R_{bfk} = -I_2$, these directions are illustrated in **Figure 8(a)**.

The base vectors of the object displacement in the null space of (49) are given by

$$\begin{aligned} Y_{Lfk2} &:= \boldsymbol{v}_\zeta \otimes X_{Lfk2} = -I_{23}^T \, {}^{o}R_{bfk} \boldsymbol{u}_1, \\ Z_{Lfk2} &:= X_{Lfk2} \otimes Y_{Lfk2} \\ &= -\begin{bmatrix} {}^{o}R_{bfk} \boldsymbol{u}_2 [{}^{Lfk}\boldsymbol{p}_o^T \, {}^{Lfk}R_{bfk} \boldsymbol{u}_1] \\ 1 \end{bmatrix} \end{aligned} \tag{52}$$

In these displacements, the length of spring s_{yk} does not change. In the case of ${}^{Lfk}R_{bfk} = -I_2$, these directions are illustrated in Figure 8(b).

C.2 Sliding contact

In the sliding contact case, the partial derivative of (21) is given by

$$\frac{\partial G_k^{fs}}{\partial s_{xk}} = 0_{3\times1}, \quad \frac{\partial G_k^{fs}}{\partial s_{yk}} = 0_{3\times1} \tag{53}$$

The partial derivative of (22) w.r.t. s_{xk} is given by

$$\frac{\partial H_k^{fs}}{\partial s_{xk}} = X'_{Lfk1} X'^T_{Lfk1} \tag{54}$$

Because (54) is positive semi-definite, the stability is enhanced when the normal component stiffness is increased. The effect appears in the following direction:

$$X'_{Lfk1} := X_{Lfk1} - \frac{\boldsymbol{u}_2^T \, {}^{\kappa fk}R_{bfk} \boldsymbol{u}_1}{{}^{Lfk}s'_{k22}} X_{Lfk}^{fd} \tag{55}$$

In the case of ${}^{\kappa fk}R_{bfk} = -I_2$, the base vectors of the object displacement in the null space of (54) are obtained by (51).

The partial derivative by s_{yk} is given by

$$\frac{\partial H_k^{fs}}{\partial s_{yk}} = X'_{\kappa ok} X'^T_{\kappa ok} \tag{56}$$

Because (56) are positive semi-definite, the stability is enhanced when the tangential stiffness is increased. The effect appears in the following direction:

$$\begin{aligned} X'_{\kappa ok} &:= \frac{1}{{}^{Lfk}s'_{k22}} \\ &\times \left\{ X_{\kappa ok2} \frac{{}^{\kappa fk}\boldsymbol{f}_k^T \boldsymbol{u}_1}{{}^{\kappa fk}\boldsymbol{p}_{\kappa ok}^T \boldsymbol{u}_1} - [{}^{o}W_{Lfk} \boldsymbol{u}_1][\boldsymbol{u}_1^T \, {}^{bfk}R_{\kappa fk} \, {}^{\kappa fk}S_k \boldsymbol{u}_2] \right\}, \\ X_{\kappa ok2} &:= {}^{o}W_{\kappa ok} \, {}^{\kappa ok}R_{bfk} \boldsymbol{u}_2 \end{aligned} \tag{57}$$

Because the derivatives (58) are not negative, the condition (20) is enhanced when the stiffness is increased.

$$\frac{\partial\, {}^{Lfk}s'_{k22}}{\partial s_{xk}} = \left\| \boldsymbol{u}_2^T \, {}^{Lfk}R_{bfk} \boldsymbol{u}_1 \right\|^2 ,\qquad \frac{\partial\, {}^{Lfk}s'_{k22}}{\partial s_{yk}} = \left\| \boldsymbol{u}_2^T \, {}^{Lfk}R_{bfk} \boldsymbol{u}_2 \right\|^2 \tag{58}$$

D. Contact force effect

We investigate effects of the normal force component ${}^{Lfk}f_{xk}$ and the tangential force component ${}^{Lfk}f_{yk}$. In this subsection, note that the treatment of the positive definiteness is reversed because ${}^{Lfk}f_{yk}$ is negative as shown in (16).

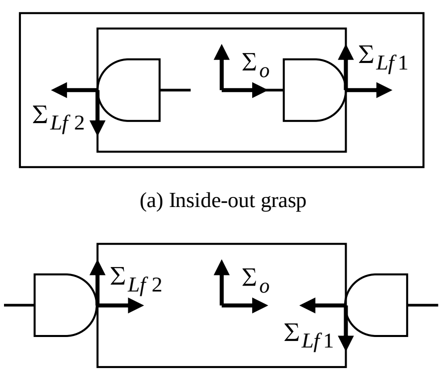
D.1 Rolling contact

In the rolling contact case, the partial derivative of (15) is given by

$$\frac{\partial G_k^{fr}}{\partial\, {}^{Lfk}f_{xk}} = {}^{o}W_{Lfk}\boldsymbol{u}_1, \quad \frac{\partial G_k^{fr}}{\partial\, {}^{Lfk}f_{yk}} = {}^{o}W_{Lfk}\boldsymbol{u}_2 \tag{59}$$

The force and moment vectors vary due to the deviation of the contact force. The partial derivative of (17) w.r.t. ${}^{Lfk}f_{xk}$ is given by

$$\frac{\partial H_k^{fr}}{\partial\, {}^{Lfk}f_{xk}} = \left\{ {}^{Lfk}\boldsymbol{p}_o^T \boldsymbol{u}_1 - \frac{1}{\kappa_{ok} + \kappa_{fk}} \right\} \boldsymbol{v}_\zeta \boldsymbol{v}_\zeta^T \tag{60}$$



(a) Inside-out grasp

(b) Outside-in grasp

Figure 9 Grasp type

If the grasp is an inside-out grasp (**Figure 9(a)**), ${}^{Lfk}\boldsymbol{p}_o^T \boldsymbol{u}_1$ is negative and (60) is negative semi-definite. Hence, the stability is enhanced when the force ${}^{Lfk}f_{xk}$ decreases (i.e., the contact force increases). If the grasp is an outside-in grasp (**Figure 9(b)**), we have ${}^{Lfk}\boldsymbol{p}_o^T \boldsymbol{u}_1 > 0$. If the condition (61) is satisfied, (60) is positive semi-definite.

$${}^{Lfk}\boldsymbol{p}_o^T \boldsymbol{u}_1 > \frac{1}{\kappa_{ok} + \kappa_{fk}} \tag{61}$$

In this case, the stability decreases when the contact force increases. The effect appears in the displacement direction $\boldsymbol{v}_\zeta$.

The effect of the tangential component force is derived as

$$\frac{\partial H_k^{fr}}{\partial\, {}^{Lfk}f_{yk}} = [{}^{Lfk}\boldsymbol{p}_o^T \boldsymbol{u}_2] \boldsymbol{v}_\zeta \boldsymbol{v}_\zeta^T \tag{62}$$

In the case of the grasp shown in **Figure 3**, (62) becomes a zero matrix.

D.2 Sliding contact

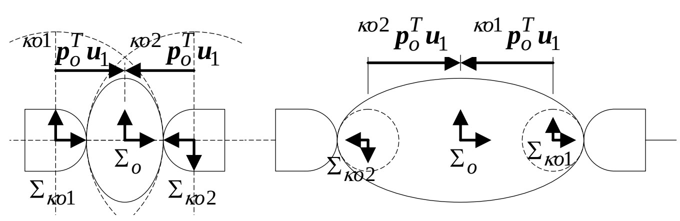
In the sliding contact case, the tangential component of the force is zero. We only investigate the effect of the normal component.

The partial derivative of (21) with respect to the normal component of the force is given by

$$\frac{\partial G_k^{fs}}{\partial\, {}^{Lfk}f_{xk}} = {}^{o}W_{Lfk}\boldsymbol{u}_1 \tag{63}$$

The force and moment vector varies due to the normal force error. The partial derivative of (22) is given by

$$\frac{\partial H_k^{fs}}{\partial\, {}^{Lfk}f_{xk}} = \frac{1}{{}^{\kappa fk}\boldsymbol{p}_{\kappa ok}^T \boldsymbol{u}_1} \left[\frac{X_{\kappa okS2}}{{}^{Lfk}s'_{k22}} \right] \left[\frac{X_{\kappa okS2}}{{}^{Lfk}s'_{k22}} \right]^T - [{}^{\kappa ok}\boldsymbol{p}_o^T \boldsymbol{u}_1] \boldsymbol{v}_\zeta \boldsymbol{v}_\zeta^T \tag{64}$$



(a) ${}^{\kappa ok}\boldsymbol{p}_o^T \boldsymbol{u}_1 > 0$ (b) ${}^{\kappa ok}\boldsymbol{p}_o^T \boldsymbol{u}_1 < 0$

Figure 10 Distance between the contact points

Positive definiteness of the first term depends on the product $\kappa_{ok}\kappa_{fk}$. If both the finger and the object are convex, the first term is positive semi-definite. In the case of convex-concave contact, the first term is negative semi-definite. If ${}^{\kappa ok}\boldsymbol{p}_o^T \boldsymbol{u}_1 > 0$ the second term is negative semi-definite (**Figure 10**). The base vector of the object's displacement in the null space of (64) is given by $Y_{\kappa okS2}$. We consider the following vectors:

$$Z_{ak} := \boldsymbol{v}_\zeta \otimes Y_{\kappa okS2}, \quad Z_{bk} := Z_{\kappa okS2} \tag{65}$$

In the direction $Z_k := a_k Z_{ak} + b_k Z_{bk}$, we have

$$Z_k^T \frac{\partial H_k^{fs}}{\partial\, {}^{Lfk}f_{xk}} Z_k = c_k \left[\frac{Z_{bk}^T \boldsymbol{v}_\zeta}{{}^{Lfk}s'_{k22}} \right]^2 \tag{66}$$

where

$$c_k := \frac{\kappa_{ok}\kappa_{fk}}{\kappa_{ok}+\kappa_{fk}} a_k^2 - [{}^{\kappa ok}\boldsymbol{p}_o^T \boldsymbol{u}_1]\, {}^{Lfk}s'^2_{k22} b_k^2 \tag{67}$$

Equation (66) is derived in Appendix C. If a_k and b_k are given from the region where c_k is positive (negative), the stability in the direction Z_k is enhanced (lowered) when the contact force decreases (increases).

From the derivative (68), ${}^{Lfk}s'_{k22}$ varies when the normal component of the finger force deviates.

$$\frac{\partial\, {}^{Lfk}s'_{k22}}{\partial\, {}^{Lfk}f_{xk}} = \frac{\kappa_{ok}\kappa_{fk}}{\kappa_{ok}+\kappa_{fk}} \tag{68}$$

When the finger force is increased, the condition (20) gets worse in case of the convex-convex contact, and the condition is improved in case of the convex-concave contact.

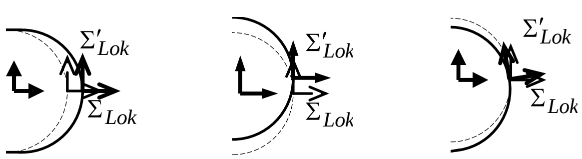
E. Local contact frame deviation effect

When the local contact frame Σ_{Lok} deviates as shown in **Figure 11**, the pose is given by

$${}^oA_{Lfk'} = {}^oA_{Lok}\, {}^{Lok}A_{Lok'}(\boldsymbol{\varepsilon}_{Lok})\, {}^{Lok}A_{Lfk} \tag{69}$$

where the pose deviation is represented by

$${}^{Lok}A_{Lok'}(\boldsymbol{\varepsilon}_{Lok}) := A_{trans}(\boldsymbol{x}_{Lok}) A_{rot}(\zeta_{Lok}) \tag{70}$$



(a) x_{Lok} (b) y_{Lok} (c) ζ_{Lok}

Figure 11 Pose deviation of the local contact frame on the object surface.

C.1. Rolling contact

Using the derivation shown in Appendix D, the partial derivative of (15) is given by

$$\frac{\partial G_k^{fr}}{\partial x_{Lok}} = -{}^{Lfk}f_{yk}\boldsymbol{v}_\zeta, \quad \frac{\partial G_k^{fr}}{\partial y_{Lok}} = {}^{Lfk}f_{xk}\boldsymbol{v}_\zeta$$
$$\frac{\partial G_k^{fr}}{\partial \zeta_{Lok}} = {}^oW_{Lfk}\Omega\, {}^{Lfk}\boldsymbol{f}_k \tag{71}$$

From these derivatives, it is observed that translational deviation of the frame affects the moment component. The rotational deviation affects not only the moment component but also the force component.

The partial derivative of (17) by x_{Lok} is given by

$$\begin{aligned}\frac{\partial H_k^{fr}}{\partial x_{Lok}} &= -\boldsymbol{v}_\zeta X_{LfkS2}^T - X_{LfkS2}\boldsymbol{v}_\zeta^T + {}^{Lfk}f_{xk}\boldsymbol{v}_\zeta\boldsymbol{v}_\zeta^T \\ &= {}^{Lfk}f_{xk}^{-1}[X_{LfkS2} - {}^{Lfk}f_{xk}\boldsymbol{v}_\zeta][X_{LfkS2} - {}^{Lfk}f_{xk}\boldsymbol{v}_\zeta]^T \\ &\quad - {}^{Lfk}f_{xk}^{-1} X_{LfkS2} X_{LfkS2}^T \end{aligned} \tag{72}$$

where $X_{LfkS2} := {}^oW_{Lfk}\, {}^{Lfk}S_k \boldsymbol{u}_2$. The first term is positive semi-definite, and the second term is negative semi-definite. When the frame deviates in the x axis, the stability decreases in the direction X_{LfkS2} and increases in the following direction.

$$X_{LfkS2} - {}^{Lfk}f_{xk}\boldsymbol{v}_\zeta \tag{73}$$

Because the derivative (72) becomes a similar form to (64), the stability deviation is investigated in a similar manner of (66). In the case of ${}^{Lfk}R_{bfk} = -I_2$, the base vectors of object's displacement in the null space of (72) are given by $\boldsymbol{v}_x$ and $\boldsymbol{v}_y$.

The partial derivative of the stiffness matrix with respect to y_{Lok} is given by

$$\frac{\partial H_k^{fr}}{\partial y_{Lok}} = \boldsymbol{v}_\zeta X_{LfkS1}^T + X_{LfkS1}\boldsymbol{v}_\zeta^T + {}^{Lfk}f_{yk}\boldsymbol{v}_\zeta\boldsymbol{v}_\zeta^T \tag{74}$$

where $X_{LfkS1} := {}^oW_{Lfk}\, {}^{Lfk}S_k \boldsymbol{u}_1$. The effects of the deviation y_{Lok} can be divided into two directions in a similar manner of (72).

$$\begin{aligned}\frac{\partial H_k^{fr}}{\partial \zeta_{Lok}} &= {}^oW_{Lfk}\Omega\, {}^{Lfk}S_k\, {}^oW_{Lfk}^T + {}^oW_{Lfk}\, {}^{Lfk}S_k \Omega^T\, {}^oW_{Lfk}^T \\ &\quad - [{}^{Lfk}\boldsymbol{f}_k^T \Omega\, {}^{Lfk}\boldsymbol{p}_o]\boldsymbol{v}_\zeta\boldsymbol{v}_\zeta^T \end{aligned} \tag{75}$$

The effect of the deviation ζ_{Lok} appears in all directions.

C.2 Sliding contact

In the sliding contact case, the partial derivatives of (21) are given by

$$\frac{\partial G_k^{fs}}{\partial x_{Lok}} = 0_{3\times 1}, \quad \frac{\partial G_k^{fs}}{\partial y_{Lok}} = {}^{Lfk}f_{xk}\boldsymbol{v}_\zeta,$$
$$\frac{\partial G_k^{fs}}{\partial \zeta_{Lok}} = {}^{o}W_{Lfk}\Omega^{Lfk}\boldsymbol{f}_k = [{}^{o}W_{Lfk}\boldsymbol{u}_2]^{Lfk}f_{xk} \qquad (76)$$

The partial derivatives of (22) with respect to x_{Lok}, y_{Lok} and ζ_{Lok} are given by

$$\frac{\partial H_k^{fs}}{\partial x_{Lok}} = \frac{\partial H_k^{fr}}{\partial x_{Lok}} + {}^{Lfk}s_{k22}\frac{\boldsymbol{v}_\zeta [X_{Lfk}^{fd}]^T + X_{Lfk}^{fd}\boldsymbol{v}_\zeta^T}{{}^{Lfk}s'_{k22}}$$
$$= {}^{Lfk}f_{xk}\left[\frac{\boldsymbol{v}_\zeta X_{\kappa ok S2}^T + X_{\kappa ok S2}\boldsymbol{v}_\zeta^T}{[{}^{\kappa fk}\boldsymbol{p}_{\kappa ok}^T\boldsymbol{u}_1]^{Lfk}s'_{k22}} + \boldsymbol{v}_\zeta\boldsymbol{v}_\zeta^T\right] \qquad (77)$$

$$\frac{\partial H_k^{fs}}{\partial y_{Lok}} = \frac{\partial H_k^{fr}}{\partial y_{Lok}} - {}^{Lfk}s_{k12}\frac{\boldsymbol{v}_\zeta [X_{Lfk}^{fd}]^T + X_{Lfk}^{fd}\boldsymbol{v}_\zeta^T}{{}^{Lfk}s'_{k22}} \qquad (78)$$

$$\frac{\partial H_k^{fs}}{\partial \zeta_{Lok}} = \frac{\partial H_k^{fr}}{\partial \zeta_{Lok}}$$
$$- \frac{{}^{o}W_{Lfk}\Omega^{Lfk}S_k\boldsymbol{u}_2[X_{Lfk}^{fd}]^T + X_{Lfk}^{fd}[{}^{o}W_{Lfk}\Omega^{Lfk}S_k\boldsymbol{u}_2]^T}{{}^{Lfk}s'_{k22}} \qquad (79)$$

where ${}^{Lfk}s_{k12} := \boldsymbol{u}_1^T\,{}^{Lfk}S_k\boldsymbol{u}_2$. The derivatives (77) and (78) can be divided into two directions in a similar manner of (72). Compared with the rolling contact case, the second terms in (77)-(79) appear due to the sliding contact. In the case of ${}^{\kappa fk}R_{bfk} = -I_2$, the second term of (78) disappears.

The effects by the deviations of Σ_{Lfk} are obtained in a similar manner of Σ_{Lok}.

F. *Contact location effect*

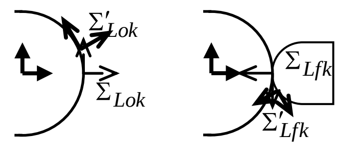
The deviation of the contact location affects the stability (**Figure 12**). The first-partial derivatives of (15), (17), (21) and (22) w.r.t. α_{ok} are obtained by

$$\frac{\partial G_k^{fc}}{\partial \alpha_{ok}} = \frac{\partial G_k^{fc}}{\partial y_{Lok}} + \kappa_{ok}\frac{\partial G_k^{fc}}{\partial \zeta_{Lok}},$$
$$\frac{\partial H_k^{fc}}{\partial \alpha_{ok}} = \frac{\partial H_k^{fc}}{\partial y_{Lok}} + \kappa_{ok}\frac{\partial H_k^{fc}}{\partial \zeta_{Lok}} \qquad (80)$$

The first-partial derivatives w.r.t. α_{fk} are obtained by

$$\frac{\partial G_k^{fc}}{\partial \alpha_{fk}} = \frac{\partial G_k^{fc}}{\partial y_{Lok}} - \kappa_{fk}\frac{\partial G_k^{fc}}{\partial \zeta_{Lok}},$$
$$\frac{\partial H_k^{fc}}{\partial \alpha_{fk}} = \frac{\partial H_k^{fc}}{\partial y_{Lok}} - \kappa_{fk}\frac{\partial H_k^{fc}}{\partial \zeta_{Lok}} \qquad (81)$$

The force and moment vector and the grasp stiffness matrix vary due to the contact location deviation. It is shown that the effects can be divided into the effects of y_{Lok} and ζ_{Lok}.



(a) α_{ok} (b) α_{fk}

Figure 12 Location deviation of the local contact frame on the object and finger surfaces.

IV. CONCLUSIONS

We analyzed the effects of the parameters' deviation on the grasp stability. It was shown that the force and moment vector and the stiffness matrix of the grasp are given by a function of contact parameters such as contact condition, local curvature, finger stiffness, and so on. The partial derivatives of the vector and the matrix w.r.t. the parameters were derived analytically. The effects were investigated from the view point of positive definiteness of the derivatives. The null spaces of the derivatives were also provided. In this space of the object pose displacement, the grasp stability is not affected by the parameters' deviation. The effects of the local contact frame deviation were also investigated. As a result of the analysis, it was shown that the stability is enhanced when the rolling contact appears, the local curvature decreases, the finger stiffness increases, and so on. The effects of the other parameters were investigated. In this paper, for generality, we discussed the case that the orientation of the finger stiffness is tilted from the contact normal and tangential directions. If the orientation is adjusted at the directions, the effects are simplified.

In our future projects, we will discuss tuning the grasp parameters based on our analysis in order to enhance the grasp stability. Finger position optimization will be attacked using the effects of the local contact frame deviation

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References

[1] H. Hanafusa and H. Asada, "Stable Prehension by a Robot Hand with Elastic Fingers", Robot Motion (ed. by M. Brady, et al.), MIT Press, pp. 323-335, 1982.

[2] V. D. Nguyen, "Constructing stable grasps", Int. J. of Robotics Research, Vol. 7, No. 3, pp. 3-16, 1988. CrossRef

[3] David J. Montana, "Contact Stability for Two-Fingered Grasps", IEEE Trans. on Robotics and Automation, Vol. 8, No. 4, pp. 421-430, 1992. CrossRef

[4] W. Howard and V. Kumar, "On the Stability of Grasped Objects", IEEE Trans. of Robotics and Automation, Vol. 12, No. 6, pp. 904-917, 1996. CrossRef

[5] E. Rimon and J. Burdick, "Mobility of Bodies in Contact. Part II: How Forces are Generated by Curvature Effects", IEEE Trans. of Robotics and Automation, Vol. 14, No. 5, pp. 696-708, 1998. CrossRef

[6] T. Yamada, S. K. Saha, N. Mimura, Y. Funahashi, "Stability Analysis of Planar Grasp with 2D-Virtual Spring Model", J. of Robotics and Mechatronics, Vol. 11, No. 4, pp. 274-282, 1999.

[7] T. Yamada, T. Taki, M. Yamada, Y. Funahashi and H. Yamamoto, "Static Stability Analysis of Spatial Grasps Including Contact Surface Geometry", Advanced Robotics, Vol. 25, No. 3, pp. 447-472, 2011. CrossRef

[8] T. Yamada, S. Yamanaka, M, Yamada, Y. Funahashi, H. Yamamoto, "Static Stability Analysis of Grasping Multiple Objects in 2D", Int. J. of Information Acquisition, Vol. 7, No. 2, pp. 119-134, 2010. CrossRef

[9] T. Yamada, T. Taki, M. Yamada, H. Yamamoto, "Static Grasp Stability Analysis of Two Spatial Objects Including Contact Surface Geometry", Trans. on Control and Mechanical Systems, Vol. 1, No. 5, pp. 218-228, 2012.

[10] T. Yamada, M. Yamada, H. Yamamoto, "Static Stability Analysis of a Single Planar Object Grasped by a Multifingered Hand", Journal of Mechanics Engineering and Automation, Vol. 2, No. 10, pp. 606-627, 2012.

[11] T. Yamada and H. Yamamoto, "Grasp Parameter Effect for Static Grasp Stability of a Single Planar Object", Proc. of SICE Annual Conference 2013, pp. 294-300, 2013

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Appendix A: Symbols

The relation of a frame Σ_b with respect to a frame Σ_a is represented by the following homogeneous transformation matrix:

$$ {}^aA_b := \begin{bmatrix} {}^aR_b & {}^a\boldsymbol{p}_b \\ 0_{1\times2} & 1 \end{bmatrix} \in \mathrm{R}^{3\times3} \tag{82} $$

where the matrix ${}^aR_b \in \mathrm{R}^{2\times2}$ represents the orientation of the frame, the vector ${}^a\boldsymbol{p}_b \in \mathrm{R}^2$ represents the position of the frame. For simplicity of the translation and rotation displacement description, the following matrices are used.

$$ A_{trans}(\boldsymbol{x}) := \begin{bmatrix} I_2 & \boldsymbol{x} \\ 0_{1\times2} & 1 \end{bmatrix}, \quad A_{rot}(\zeta) := \begin{bmatrix} \mathrm{Rot}(\zeta) & 0_{2\times1} \\ 0_{1\times2} & 1 \end{bmatrix}, \\ \mathrm{Rot}(\zeta) := \begin{bmatrix} \cos\zeta & -\sin\zeta \\ \sin\zeta & \cos\zeta \end{bmatrix} \tag{83} $$

In two dimensional space, the vector $\boldsymbol{x} = [x, y]^T$ and the scalar ζ represent the translation component and the rotation component, respectively. The matrix I_n is an n-by-n identity matrix.

We also define the following symbols:

$$ {}^aW_b := \begin{bmatrix} {}^aR_b \\ {}^a\boldsymbol{p}_b \otimes {}^aR_b \end{bmatrix} \in \mathrm{R}^{3\times2}, \quad \Omega := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \mathrm{Rot}\left(\frac{\pi}{2}\right), \\ I_{23} := [I_2, 0_{2\times1}], \quad \boldsymbol{u}_1 := [1,0]^T, \quad \boldsymbol{u}_2 := [0,1]^T, \\ \boldsymbol{v}_x := [1,0,0]^T, \quad \boldsymbol{v}_y := [0,1,0]^T, \quad \boldsymbol{v}_\zeta := [0,0,1]^T \tag{84} $$

The symbol $\otimes$ is represented for the cross product in 2D planar space or configuration space.

The vector $\boldsymbol{\varepsilon}_o = [\boldsymbol{x}_o^T, \zeta_o]^T$ is the object pose displacement. Taylor series of the potential function is given by

$$ U(\boldsymbol{\varepsilon}_o) = U(0) + \boldsymbol{\varepsilon}_o^T G + \tfrac{1}{2}\boldsymbol{\varepsilon}_o^T H \boldsymbol{\varepsilon}_o + \cdots \tag{85} $$

where

$$ G := \left.\frac{\partial U(\boldsymbol{\varepsilon}_o)}{\partial \boldsymbol{\varepsilon}_o}\right|_0, \quad H := \left.\frac{\partial^2 U(\boldsymbol{\varepsilon}_o)}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\varepsilon}_o^T}\right|_0 \tag{86} $$

Appendix B: Derivation of (39)

The vector $Y_{\kappa fk}$ is calculated as follows:

$$ \begin{aligned} Y_{\kappa fk} &= \boldsymbol{v}_\zeta \otimes [{}^oW_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2 - {}^{\kappa fk}f_{xk}\boldsymbol{v}_\zeta] \\ &= \boldsymbol{v}_\zeta \otimes [{}^oW_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2] \\ &= \begin{bmatrix} \Omega & 0_{2\times1} \\ 0_{1\times2} & 0 \end{bmatrix} \begin{bmatrix} {}^oR_{\kappa fk} \\ {}^o\boldsymbol{p}_{\kappa fk} \otimes {}^oR_{\kappa fk} \end{bmatrix} {}^{\kappa fk}S_k \boldsymbol{u}_2 \\ &= \begin{bmatrix} I_2 \\ 0_{1\times2} \end{bmatrix} \Omega\, {}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2 = I_{23}^T \Omega\, {}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2 \end{aligned} \tag{87} $$

The vector $Z_{\kappa fk}$ is calculated as follows:

$$ \begin{aligned} Z_{\kappa fk} &= [{}^oW_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2 - {}^{\kappa fk}f_{xk}\boldsymbol{v}_\zeta] \otimes [I_{23}^T \Omega\, {}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2] \\ &= \begin{bmatrix} {}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2 \\ {}^o\boldsymbol{p}_{\kappa fk} \otimes {}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2 - {}^{\kappa fk}f_{xk} \end{bmatrix} \otimes \begin{bmatrix} \Omega\, {}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2 \\ 0_{1\times2} \end{bmatrix} \\ &= \begin{bmatrix} \Omega[\Omega\, {}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2][{}^o\boldsymbol{p}_{\kappa fk} \otimes {}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2 - {}^{\kappa fk}f_{xk}] \\ [{}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2] \otimes [\Omega\, {}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2] \end{bmatrix} \\ &= \begin{bmatrix} [{}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2][{}^{\kappa fk}\boldsymbol{p}_o \otimes {}^{\kappa fk}S_k \boldsymbol{u}_2 + {}^{\kappa fk}f_{xk}] \\ [\Omega\, {}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2]^T [\Omega\, {}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2] \end{bmatrix} \\ &= \begin{bmatrix} [{}^oR_{\kappa fk}\,{}^{\kappa fk}S_k \boldsymbol{u}_2][{}^{\kappa fk}f_{xk} - {}^{\kappa fk}\boldsymbol{p}_o^T \Omega\, {}^{\kappa fk}S_k \boldsymbol{u}_2] \\ \left\| {}^{\kappa fk}S_k \boldsymbol{u}_2 \right\|^2 \end{bmatrix} \end{aligned} \tag{88} $$

APPENDIX C: DERIVATION OF (66)

Equation (66) is obtained from the following procedure:

$$Z_k^T \frac{\partial H_k^{fs}}{\partial^{Lfk} f_{xk}} Z_k$$
$$= \frac{[Z_k^T X_{\kappa ok S2}]^2}{[{}^{\kappa fk}\boldsymbol{p}_{\kappa ok}^T \boldsymbol{u}_1]^{Lfk} s_{k22}'^2} - [{}^{\kappa ok}\boldsymbol{p}_o^T \boldsymbol{u}_1][Z_k^T \boldsymbol{v}_\zeta]^2$$
$$= a_k^2 \frac{[Z_{ak}^T X_{\kappa ok S2}]^2}{[{}^{\kappa fk}\boldsymbol{p}_{\kappa ok}^T \boldsymbol{u}_1]^{Lfk} s_{k22}'^2} - b_k^2 [{}^{\kappa ok}\boldsymbol{p}_o^T \boldsymbol{u}_1][Z_{bk}^T \boldsymbol{v}_\zeta]^2$$
$$= \left\{ \frac{a_k^2}{{}^{\kappa fk}\boldsymbol{p}_{\kappa ok}^T \boldsymbol{u}_1} - [{}^{\kappa ok}\boldsymbol{p}_o^T \boldsymbol{u}_1]^{Lfk} s_{k22}'^2 b_k^2 \right\} \left\{ \frac{Z_{bk}^T \boldsymbol{v}_\zeta}{{}^{Lfk} s_{k22}'} \right\}^2$$
$$= \left\{ \frac{\kappa_{ok}\kappa_{fk}}{\kappa_{ok}+\kappa_{fk}} a_k^2 - [{}^{\kappa ok}\boldsymbol{p}_o^T \boldsymbol{u}_1]^{Lfk} s_{k22}'^2 b_k^2 \right\} \left\{ \frac{Z_{bk}^T \boldsymbol{v}_\zeta}{{}^{Lfk} s_{k22}'} \right\}^2 \quad (89)$$

APPENDIX D: DERIVATION OF (71) AND (80)

From (69) and (70), we have

$$\frac{\partial^o A_{Lfk'}}{\partial x_{Lok}} = {}^o A_{Lok} \left\{ \frac{\partial^{Lok} A_{Lok'}(\boldsymbol{\varepsilon}_{Lok})}{\partial x_{Lok}} {}^{Lok} A_{Lok'}^{-1}(\boldsymbol{\varepsilon}_{Lok}) \right\}$$
$$\times {}^{Lok} A_{Lok'}(\boldsymbol{\varepsilon}_{Lok}) {}^{Lok} A_{Lfk}$$
$$= {}^o A_{Lok} \begin{bmatrix} 0_{2\times2} & \boldsymbol{u}_1 \\ 0_{1\times2} & 0 \end{bmatrix} {}^{Lok} A_{Lok'}(\boldsymbol{\varepsilon}_{Lok}) {}^{Lok} A_{Lfk} \quad (90)$$

At the initial condition, we have

$$\left.\frac{\partial^o A_{Lfk'}}{\partial x_{Lok}}\right|_0 = {}^o A_{Lok} \begin{bmatrix} 0_{2\times2} & \boldsymbol{u}_1 \\ 0_{1\times2} & 0 \end{bmatrix} {}^{Lok} A_{Lfk} = \begin{bmatrix} 0_{2\times2} & {}^o R_{Lok}\boldsymbol{u}_1 \\ 0_{1\times2} & 0 \end{bmatrix} \quad (91)$$

Therefore we have

$$\frac{\partial^o W_{Lfk}}{\partial x_{Lok}} = \boldsymbol{v}_\zeta \left[\frac{\partial^o \boldsymbol{p}_{Lfk}}{\partial x_{Lok}} \otimes \right] {}^o R_{Lfk} + \left[\begin{matrix} I_2 \\ {}^o\boldsymbol{p}_{Lfk} \otimes \end{matrix} \right] \frac{\partial^o R_{Lfk}}{\partial x_{Lok}}$$
$$= \boldsymbol{v}_\zeta [({}^o R_{Lok}\boldsymbol{u}_1) \otimes {}^o R_{Lfk}] = \boldsymbol{v}_\zeta \boldsymbol{u}_2^T {}^{Lok} R_{Lfk}$$
$$= -\boldsymbol{v}_\zeta \boldsymbol{u}_2^T \quad (92)$$

and

$$\frac{\partial G_k^{fr}}{\partial x_{Lok}} = \frac{\partial^o W_{Lfk}}{\partial x_{Lok}} {}^{Lfk}\boldsymbol{f}_k = -{}^{Lfk} f_{yk} \boldsymbol{v}_\zeta \quad (93)$$

In the similar manner of x_{Lok}, we have

$$\frac{\partial^o W_{Lfk}}{\partial y_{Lok}} = \boldsymbol{v}_\zeta [({}^o R_{Lok}\boldsymbol{u}_2) \otimes {}^o R_{Lfk}] = -\boldsymbol{v}_\zeta \boldsymbol{u}_1^T {}^{Lok} R_{Lfk}$$
$$= \boldsymbol{v}_\zeta \boldsymbol{u}_1^T \quad (94)$$

The partial derivative by ζ_{Lok} is obtained by the following procedure:

$$\frac{\partial^o A_{Lfk'}}{\partial \zeta_{Lok}} = {}^o A_{Lok} {}^{Lok} A_{Lok'}(\boldsymbol{\varepsilon}_{Lok}) \begin{bmatrix} \Omega & 0_{2\times1} \\ 0_{1\times2} & 0 \end{bmatrix} {}^{Lok} A_{Lfk},$$
$$\left.\frac{\partial^o A_{Lfk'}}{\partial \zeta_{Lok}}\right|_0 = {}^o A_{Lok} \begin{bmatrix} \Omega & 0_{2\times1} \\ 0_{1\times2} & 0 \end{bmatrix} {}^{Lok} A_{Lfk}$$
$$= \begin{bmatrix} {}^o R_{Lok}\Omega {}^{Lok} R_{Lfk} & {}^o R_{Lok}\Omega {}^{Lok}\boldsymbol{p}_{Lfk} \\ 0_{1\times2} & 0 \end{bmatrix} \quad (95)$$
$$= \begin{bmatrix} {}^o R_{Lfk}\Omega & 0_{2\times1} \\ 0_{1\times2} & 0 \end{bmatrix},$$
$$\frac{\partial^o W_{Lfk}}{\partial \zeta_{Lok}} = \left[\begin{matrix} I_2 \\ {}^o\boldsymbol{p}_{Lfk} \otimes \end{matrix} \right] {}^o R_{Lfk}\Omega = {}^o W_{Lfk}\Omega$$

The partial derivative by α_{ok} is obtained by the following procedure:

$$\frac{\partial^o A_{Cfk}(\alpha_{ok})}{\partial \alpha_{ok}} = {}^o A_{\kappa ok} A_{rot}(\kappa_{ok}\alpha_{ok}) \begin{bmatrix} \kappa_{ok}\Omega & 0_{2\times1} \\ 0_{1\times2} & 0 \end{bmatrix} {}^{\kappa ok} A_{Lfk},$$
$$\left.\frac{\partial^o A_{Lfk}(\alpha_{ok})}{\partial \alpha_{ok}}\right|_0 = \begin{bmatrix} \kappa_{ok} {}^o R_{Lfk}\Omega & -\kappa_{ok} {}^o R_{Lfk}\Omega {}^{Lfk}\boldsymbol{p}_{\kappa ok} \\ 0_{1\times2} & 0 \end{bmatrix}$$
$$= \begin{bmatrix} \kappa_{ok} {}^o R_{Lfk}\Omega & -{}^o R_{Lfk}\boldsymbol{u}_2 \\ 0_{1\times2} & 0 \end{bmatrix}, \quad (96)$$
$$\frac{\partial^o W_{Lfk}}{\partial \alpha_{ok}} = \boldsymbol{v}_\zeta \left[\frac{\partial^o \boldsymbol{p}_{Lfk}}{\partial \alpha_{ok}} \otimes \right] {}^o R_{Lfk} + \left[\begin{matrix} I_2 \\ {}^o\boldsymbol{p}_{Lfk} \otimes \end{matrix} \right] \frac{\partial^o R_{Lfk}}{\partial \alpha_{ok}}$$
$$= \frac{\partial^o W_{Lfk}}{\partial y_{Lok}} + \kappa_{ok} \frac{\partial^o W_{Lfk}}{\partial \zeta_{Lok}}$$