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The Journal of Instrumentation, Automation and Systems

Simultaneous Trajectory Tracking and Formation Keeping for a Group of Unicycle Mobile Robots

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Abstract—This paper discusses the design of a control algorithm for a group of unicycles that is able to simultaneously track individual references and keep a certain formation with other unicycles. The algorithm extends the concept of individual trajectory tracking designed using dynamic feedback linearization. The extension is done by introducing coupling gains between the robots, so that formation keeping can be achieved. The algorithm is validated in real-time experiments. Using a root-mean-square error-like indicator, the influence of communication topologies on the performance of the control algorithm is investigated. It is shown that communication topologies and gains are the most influencing parameters.

Keywords—Trajectory tracking, formation keeping, unicycle mobile robots, dynamic feedback linearization, coupling.

I. Introduction

A motion coordination of multiple robots has been actively studied in the field of cooperative control, see e.g. [1]-[3]. Assignment of feasible formations, making a certain formation, maintaining a certain formation shape, and switching between formations are some examples of formation related tasks. The problem of formation control of unicycles has been widely investigated; see for examples [1]-[7].

Different control method such as Lyapunov technique, feedback linearization, and Model Predictive Control were used to solve classical formation control approaches, namely the leader-follower problem, the behavior-based, and the virtual-structure. The work in [6] considered saturation on the control signals, while the work in [7] considers the practical aspect of using non-linear MPC to control a group of robots. In [1] and [2], the formation is defined as how a member keeps a certain distance and relative angle with respect to other robots. In [4]-[6], the formation is defined as the relative distances between the robots. The formation can be either time-varying or non-time varying.

In [4]-[6], the problem of motion coordination of unicycle agents is studied. The scheme in [5] achieves coordination of two agents under certain excitation properties of the reference trajectories; no saturation constraints on the control signals are considered. The scheme in [6] relaxes the conditions on the excitation properties of the reference trajectories and achieves coordination of multiple agents under saturation constraints on inputs. An algorithm for collision avoidance proposed in [6] is applicable with both control schemes. In [5] and [6], the coordination is established by mutual coupling of motion controllers of the individual agents. These couplings are introduced at the level of the tracking errors. Although each scheme practically leads to some form of formation control, none is derived starting from some explicit formation control goal. The mathematical objective of both schemes is, in fact, trajectory tracking, so formation control is only an implicit consequence of mutual coupling of the tracking controllers of the interacting agents.

Furthermore, in the problem of motion coordination, it is important to generate a performance measure that can be used to evaluate the performance of the control algorithms. The work of Tanner in [8]-[11] illustrates how performance of formation control is formulated by means of the input signals of the leader and information topology between the robots. These works combine the feedback linearization technique for the control design and input-to-state stability (ISS) concept to formulate a performance measure. The results show that the performance measure, although depending on the control gain values, gives an analytical prediction on how good the group of robots maintains the formation.

In addition, the work in [12] provides a performance index that shows the discrepancy between the desired and the actual formation. This index measures the difference between desired and actual distances between all pairs of robots in the formation. In particular, the index measures the geometric of the formation but with the possibility to have a rotation or a reflection (a mirror image) of the formation shape.

Motivated by the importance of having an efficient time-varying motion coordination control algorithm and a performance evaluation, this paper presents a coordination control algorithm, and its performance measure, for a group of unicycles, which is able to simultaneously track individual reference trajectories and keep a certain formation with other robots. The algorithm extends the trajectory tracking controller for a single robot designed using dynamic feedback linearization as proposed in [13]. The stability is proven using a theorem of interconnected systems.

The contributions of this paper are the following. Firstly, a controller that is able to simultaneously track individual

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This paper was submitted on May 10, 2015; revised on June 07, 2015; and accepted on June 09, 2015.

references and keep a certain spatial formation is proposed. The tasks can be achieved efficiently by sharing individual tracking errors within the group. Secondly, by means of experiments, the influence of communication topologies between the robots on the performance is analyzed using a root-mean-square error-like indicator.

The rest of this paper is organized as follows. Section II gives mathematical background for this paper. Section III provides the design of the control algorithm and the stability proof. Section IV shows the experimental results and performance analysis of the control algorithm. Section V gives the conclusions of this work and some ideas for future works.

II. Mathematical Preliminaries

This section presents a short mathematical background on kinematic model of unicycle mobile robots and stability of interconnected systems.

A. Kinematic model of unicycle mobile robots

In this paper, a nonholonomic kinematic model of unicycles is considered as the mobile robot model, see **Figure 1**.

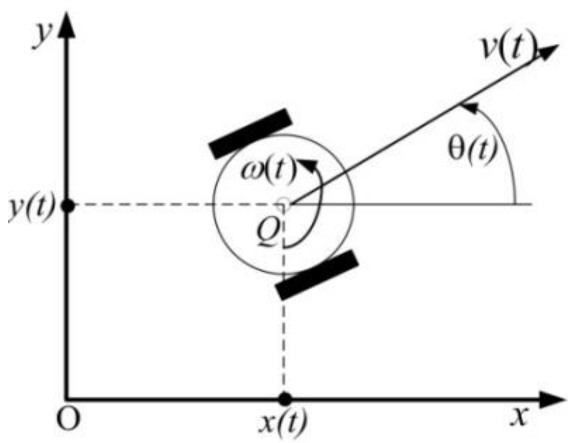


Figure 1 Schematic representation of unicycle mobile robots

The states of the system are $[x(t) \quad y(t) \quad \theta(t)]^T$, where $x(t)$, $y(t)$, and $\theta(t)$ are the position (in x and y direction) and orientation of the robot. The posture kinematic model of the unicycle is given as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} \tag{1}$$

where $x(t)$ and $\omega(t)$ are the forward/translational velocity and rotational/steering velocity respectively.

B. Stability of interconnected systems

Consider the interconnected system:

$$\dot{x}_i = f_i(t, x_i) + g_i(t, x), \qquad i = 1,2,\dots,m \tag{2}$$

where $x_i \in R^{n_i}, n_1 + \cdots + n_m = n$ and $x = [x_1^T \dots . x_m^T]^T$.

The function

$$V(t,x) = \sum_{i=1}^{m} d_i V_i(t, x_i), d_i > 0 \tag{3}$$

is a composite Lyapunov function for the collection of the m isolated subsystems for all values of positive d_i. Suppose that for $i = 1,2,\dots,m$, $V_i(t, x_i)$ satisfies

$$\frac{\partial V_i}{\partial t} + \frac{\partial V_i}{\partial x_i} f_i(t, x_i) \le -\alpha_i \varphi_i(x_i) \tag{4}$$

$$\left\| \frac{\partial V_i}{\partial x_i} \right\| \le \beta_i \varphi_i(x_i), \tag{5}$$

for all $t \ge 0$ and $\|x\| < r$ for some positive constants α_i and β_i, where $\varphi_i: R^{n_i} \to R$ are positive definite and continuous functions. Furthermore, suppose that the interconnection $g_i(t,x)$ satisfies the bound

$$\|g_i(t,x)\| \le \sum_{j=1}^{m} \gamma_{ij} \varphi_j(x_j) \tag{6}$$

for all $t \ge 0$ and $\|x\| < r$ for some nonnegative constants γ_{ij}. In addition, define an $m \times m$ matrix S whose elements are defined by

$$s_{ij} = \begin{cases} \alpha_i - \beta_i \gamma_{ii}, & i = j \\ -\beta_i \gamma_{ij}, & i \ne j \end{cases} \tag{7}$$

and S is required to be an M-matrix.

To satisfy an M-matrix condition, it is sufficient to have the leading principal minors of S positive or det $S > 0$.

Theorem 1 [14] *Consider system* (2) *and suppose there are positive definite decrescent Lyapunov functions* $V_i(t, x_i)$ *that satisfy* (4) *and* (5) *and that* $g_i(t,x)$ *satisfies* (6) *for all* $t \ge 0$ and $\|x\| < r$. *Suppose that the matrix S defined by* **(7)** *is an M-matrix. Then, the origin is uniformly asymptotically stable. Moreover, if all the assumptions hold globally and* $V_i(t, x_i)$ *are radially unbounded, the origin is globally uniformly asymptotically stable.*

III. Control Design

A. Trajectory tracking controller of a single robot

In [13], by introducing a new state, $\varsigma_i = v_i$ and new inputs $u_{i1} = \dot{\varsigma}_i$, $u_{i1} = \omega_i$, the kinematic model of unicycle *i* (1) can be transformed into (neglecting *t*):

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \\ \dot{\varsigma}_i \end{bmatrix} = \begin{bmatrix} \varsigma_i \cos\theta_i \\ \varsigma_i \sin\theta_i \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_{i1} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u_{i2} \tag{8}$$

Choosing output $\mathbf{h}_i = [x_i \quad y_i]^{\mathrm{T}}$, the second order derivative of $\mathbf{h}_i$ is

$$\begin{bmatrix}\ddot{x}_i\\ \ddot{y}_i\end{bmatrix} = \begin{bmatrix}\cos\theta_i & -\varsigma_i\sin\theta_i\\ \sin\theta_i & \varsigma_i\cos\theta_i\end{bmatrix}\begin{bmatrix}u_{i1}\\ u_{i2}\end{bmatrix} \tag{9}$$

As long as $\varsigma_i \neq 0$, the extended dynamic model of the unicycle is input-output feedback linearizable into $\ddot{\mathbf{h}}_i = \begin{bmatrix}\ddot{x}_i\\ \ddot{y}_i\end{bmatrix} = \begin{bmatrix}w_{i1}\\ w_{i2}\end{bmatrix} = \mathbf{w}_i$, using the control signals given by

$$\begin{bmatrix}u_{i1}\\ u_{i2}\end{bmatrix} = \begin{bmatrix}\cos\theta_i & \sin\theta_i\\ \frac{-\sin\theta_i}{\varsigma_i} & \frac{\cos\theta_i}{\varsigma_i}\end{bmatrix}\begin{bmatrix}w_{i1}\\ w_{i2}\end{bmatrix} \tag{10}$$

The resulting dynamic compensator can be formulated as follows:

$$\dot{\varsigma}_i = w_{i1}\cos\theta_i + w_{i2}\sin\theta_i, \tag{11a}$$

$$v_i = \varsigma_i, \tag{11b}$$

$$\omega_i = \frac{-w_{i1}\sin\theta_i + w_{i2}\cos\theta_i}{v_i} \tag{12}$$

The above dynamic compensator has potential singularity at $v_i = \varsigma_i = 0$, i.e. when the unicycle is not rolling. This singularity is structural for nonholonomic system [15].

Assume that the reference forward velocity is positive, i.e. $v_{ri} = \sqrt{\dot{x}_{ri}^2 + \dot{y}_{ri}^2} > 0$, a proportional-derivative (PD) controller for exponentially tracking the reference trajectories can be achieved by setting:

$$\begin{bmatrix}w_{i1}\\ w_{i2}\end{bmatrix} = \begin{bmatrix}\ddot{x}_{ri} + k_i^{dx}(\dot{x}_{ri} - \dot{x}_i) + k_i^{px}(x_{ri} - x_i)\\ \ddot{y}_{ri} + k_i^{dy}(\dot{y}_{ri} - \dot{y}_i) + k_i^{py}(y_{ri} - y_i)\end{bmatrix} \tag{13}$$

with the PD gains $k_i^{dx}, k_i^{px}, k_i^{dy}, k_i^{py} > 0$. By choosing $\mathbf{e}_i = \begin{bmatrix}e_{xi}\\ e_{yi}\end{bmatrix} \triangleq \begin{bmatrix}x_{ri} - x_i\\ y_{ri} - y_i\end{bmatrix}$, one can simply obtain second order closed loop error dynamics of unicycle i on the equivalent linear and decoupled systems given in (13).

In [13], a sufficient condition to guarantee that no singularity occurs is proposed. One important aspect of the theorem is that to obtain exact trajectory tracking for a matched initial posture of the robot, i.e. $x_i^0 = x_{ri}^0$, $y_i^0 = y_{ri}^0$, $\theta_i^0 = \theta_{ri}^0$ (or $\theta_i^0 = \theta_{ri}^0 + \pi$), the dynamic compensator should be correctly initialized at $\varsigma_i^0 = v_{ri}^0$.

B. An extension to formation keeping of a group of unicycles

Let m be the number of robots in the formation. A spatial formation pattern of the group of robots is defined as

$$F(t) = \{\mathbf{h}_{r1}(t), \mathbf{h}_{r2}(t), \dots, \mathbf{h}_{rm}(t)\} \tag{14}$$

where $\mathbf{h}_{ri}(t)$ are the desired Cartesian coordinates of the center of the i-th robot with respect to time.

A class of formation control is defined as the requirement of the group of robots to follow $F(t)$, while individually, robots have to follow their own reference trajectories. In this type of formation, the problem typically occurs when perturbations are introduced to the group. During the transition to recover from perturbations, there will be conflicting objectives. On the one hand, the group will try to recover the desired spatial pattern $F(t)$. On the other hand, by trying to recover the formation, individual robots maybe required to leave their individual reference trajectory $\mathbf{h}_{ri}(t)$, which may be undesirable.

Define ξ_i as

$$\xi_i \triangleq \begin{bmatrix}x_{ri} - x_i\\ y_{ri} - y_i\\ \dot{x}_{ri} - \dot{x}_i\\ \dot{y}_{ri} - \dot{y}_i\end{bmatrix} = \begin{bmatrix}\mathbf{e}_i\\ \dot{\mathbf{e}}_i\end{bmatrix} \tag{15}$$

the simultaneous trajectory tracking and formation keeping is formulated as the problem to render individual tracking errors $\xi_i \to 0, \forall i \in \{1,2,\dots,m\}$ as $t \to \infty$.

To achieve simultaneous trajectory tracking and formation keeping, the PD controller in (13) is modified by introducing coupling terms. Thus, $\mathbf{w}_i$ becomes:

$$\mathbf{w}_i = \begin{bmatrix}\ddot{x}_{ri} + k_i^{dx}\dot{e}_{xi} + k_i^{px}e_{xi} + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{dx}(\dot{e}_{xi} - \dot{e}_{xj}) + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{px}(e_{xi} - e_{xj})\\ \ddot{y}_{ri} + k_i^{dy}\dot{e}_{yi} + k_i^{py}e_{yi} + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{dy}(\dot{e}_{yi} - \dot{e}_{yj}) + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{px}(e_{yi} - e_{yj})\end{bmatrix} \tag{16}$$

where $k_{ij}^{\rho}, \rho \in \{px, dx, py, dy\}$ are the coupling gains.

The PD controller in (16) basically consists of two parts. The part related to k_i^{ρ} is responsible for trajectory tracking, while the part related to k_{ij}^{ρ} is responsible for adding robustness to formation keeping. Depending on the choices of k_i^{ρ} and k_{ij}^{ρ}, the group of robots can achieve simultaneous trajectory tracking and formation keeping or pure trajectory tracking or pure formation keeping.

Substituting (16) into (13), we obtain

$$\begin{bmatrix}\ddot{x}_i\\ \ddot{y}_i\end{bmatrix} = \begin{bmatrix}\ddot{x}_{ri} + k_i^{dx}\dot{e}_{xi} + k_i^{px}e_{xi} + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{dx}(\dot{e}_{xi} - \dot{e}_{xj}) + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{px}(e_{xi} - e_{xj})\\ \ddot{y}_{ri} + k_i^{dy}\dot{e}_{yi} + k_i^{py}e_{yi} + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{dy}(\dot{e}_{yi} - \dot{e}_{yj}) + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{px}(e_{yi} - e_{yj})\end{bmatrix} \tag{17}$$

Thus, the closed loop error dynamics of the simultaneous trajectory tracking and formation keeping of m unicycles can be formulated as follows:

$$\begin{bmatrix}\ddot{e}_{xi} + k_i^{dx}\dot{e}_{xi} + k_i^{px}e_{xi} + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{dx}(\dot{e}_{xi} - \dot{e}_{xj}) + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{px}(e_{xi} - e_{xj})\\ \ddot{e}_{yi} + k_i^{dy}\dot{e}_{yi} + k_i^{py}e_{yi} + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{dy}(\dot{e}_{yi} - \dot{e}_{yj}) + \sum_{\substack{j=1\\ j\neq i}}^{m} k_{ij}^{px}(e_{yi} - e_{yj})\end{bmatrix} = 0 \tag{18}$$

For the closed-loop error dynamics (18), the following theorem holds.

Theorem 2 Consider the second order dynamics of m unicycle robots described in (18). If $k_i^{\rho x}, k_i^{dx}, k_i^{\rho y}, k_i^{dx} > 0$ and $k_{ij}^{\rho x}, k_{ij}^{dx}, k_{ij}^{\rho y}, k_{ij}^{dx} \geq 0$, then the controller formulated in (16) (combined with the dynamic compensator in (11)-(12)) renders the origin of (18), $\forall i \in \{1,2,\dots,m\}$ asymptotically stable.

Proof. The stability proof follows Theorem 1 on the stability of interconnected systems. Using ξ_i in (15), (17) can be written as:

$$\dot{\xi}_i = -A_i\xi_i + \sum_{\substack{j=1 \\ j\neq 1}}^{m} B_{ij}\xi_j, \qquad i \in \{1,2,\dots,m\} \tag{19}$$

With

$$A_i = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ \left(k_i^{px} + \sum_{\substack{j=1 \\ j\neq i}}^{m} k_{ij}^{px}\right) & 0 & \left(k_i^{px} + \sum_{\substack{j=1 \\ j\neq i}}^{m} k_{ij}^{px}\right) & 0 \\ 0 & \left(k_i^{px} + \sum_{\substack{j=1 \\ j\neq i}}^{m} k_{ij}^{px}\right) & 0 & \left(k_i^{px} + \sum_{\substack{j=1 \\ j\neq i}}^{m} k_{ij}^{px}\right) \end{bmatrix} \tag{20}$$

$$B_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ k_{ij}^{px} & 0 & k_{ij}^{py} & 0 \\ 0 & k_{ij}^{dx} & 0 & k_{ij}^{dx} \end{bmatrix} \tag{21}$$

Define $f_i(\xi_i) = -A_i\xi_i$ and $g_i(\xi_i) = \sum_{\substack{j=1 \\ j\neq i}}^{m} B_{ij}\xi_j$. Using $V_i(\xi_i) = 0.5\,\xi_i^T\xi_i$ as a Lyapunov candidate for the i-th isolated subsystems, we obtain

$$\begin{aligned} \frac{\partial V_i}{\partial \xi_i} f_i(\xi_i) &= -A_i\xi_i^T\xi_i \\ &\leq -\|A_i\|\xi_i^T\xi_i \\ &\leq -\alpha_i\varphi_i^2(\xi_i) \end{aligned} \tag{22}$$

with $\alpha_i = \|A_i\|$, $\varphi_i(\xi_i) = \|\xi_i\|$, and

$$\begin{aligned} \left\|\frac{\partial V_i}{\partial \xi_i}\right\| &\leq \|\xi_i\| \\ &\leq \beta_i\varphi_i(\xi_i) \end{aligned} \tag{23}$$

with $\beta_i = 1$. Thus $V_i(\xi_i)$ satisfies (4) and (5). Furthermore, the interconnection terms satisfies the following inequality:

$$\begin{aligned} \|g_i(\xi)\| &\leq \sum_{\substack{j=1 \\ j\neq 1}}^{m} \|B_{ij}\|\|\xi_j\| \\ &\leq \sum_{\substack{j=1 \\ j\neq i}}^{m} \gamma_{ij}\,\varphi_j(\xi_j) \end{aligned} \tag{24}$$

Thus, $g_i(\xi)$ satisfies **(6)** with $\varphi_j(\xi_j) = \|\xi_j\|$, $\gamma_{ii} = 0$ and $\gamma_{ij} = \|B_{ij}\|$ for $i \neq j$. Matrix S is then can be formulated as follows:

$$s_{ij} = \begin{cases} \|A_i\| & \text{for } i = j \\ -\|B_{ij}\| & \text{for } i \neq j \end{cases} \tag{25}$$

with A_i and B_{ij} given in (20) and (21) respectively such that

$$S = \begin{bmatrix} \|A_1\| & \cdots & -\|B_{1m}\| \\ \vdots & \ddots & \vdots \\ -\|B_{m1}\| & \cdots & \|A_m\| \end{bmatrix} \tag{26}$$

The equilibrium point, in this case $\xi_i = 0$, is asymptotically stable if S is an M matrix. The M matrix condition can be interpreted as a requirement that the diagonal elements of S be "larger as a whole" than the off-diagonal elements. It can be shown that diagonally dominant matrices with non-positive off-diagonal elements are M matrices [14].

In this particular case, it is straightforward to find that the S in (25) is an M matrix since by the construction of matrices A_i and B_{ij}, the main diagonal components of S are always larger than the sum of the off diagonal component. Thus, the equilibrium point of $\xi_i, i \in \{1,2,\dots,m\}$ is asymptotically stable provided that S satisfies the M matrix condition. This means that the interconnection with other robots satisfies the M matrix requirement. This completes the proof. ■

It is to be noted that the simultaneous trajectory tracking and formation keeping is developed from a single trajectory tracking control problem. Thus, the two objectives, i.e. simultaneous tracking and formation keeping, can be achieved by means of tracking only. The introduction of coupling gains increases the robustness against perturbation in keeping the formation. It is sufficient to have $k_i^{\rho x}, k_i^{dx}, k_i^{\rho y}, k_i^{dx} > 0$, while the coupling gain can be chosen freely (as long as they are not negative) to achieve simultaneous tracking and formation keeping. This condition is easy to verify by modifying the S matrix in (26) into summation of two matrices. One of them is a diagonal matrix with $\|A_{11}\|$, $\|A_{21}\|$, ..., $\|A_{m1}\|$ as its components. In this way, it is easy to prove that the asymptotically stable equilibrium point can be achieved by means of trajectory tracking only.

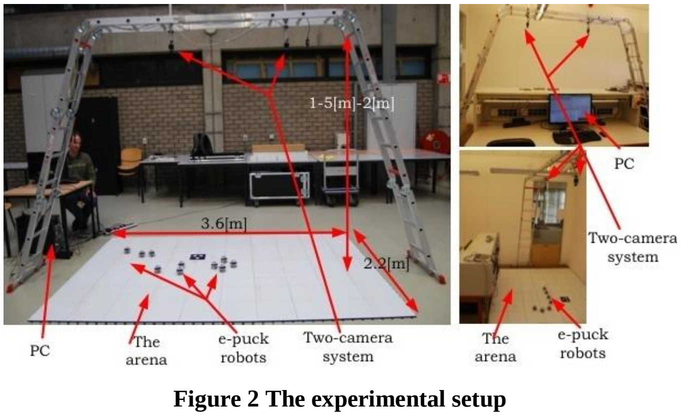


Figure 2 The experimental setup

IV. Experimental Results

A. *Experimental setup*

The experiments are conducted at the Laboratory of the Dynamics and Control Group, Department of Mechanical Engineering, Eindhoven University of Technology, The Netherlands. The setup is depicted in **Figure 2**.

We use mobile robots, model e-puck [16], two-camera system as a localization device for getting the position and orientation of all robots, and a PC. The PC generates robot trajectories, processes camera images to get actual pose of the robots, and runs control algorithms. The PC sends the control velocities to the robots via a BlueTooth protocol.

B. *Results and analysis*

The following control gains are used: $k_i^{px} = k_i^{py} = 1$, $k_i^{dx} = k_i^{dy} = 2$, $k_{ij}^{px} = k_{ij}^{py} = 4$, $k_{ij}^{dx} = k_{ij}^{dy} = 1$. The individual trajectory tracking gains, k_i^{px}, k_i^{py} and k_i^{dx}, k_i^{dy} are chosen so that a (linear) second-order system (of the error dynamics given in (18)) with critically damped behavior is obtained. Basically it requires $k_i^{d\epsilon} = 2\sqrt{k_i^{p\epsilon}}, \epsilon \in \{x, y\}$ [17].

In all experiments, the forward velocity (dynamic compensator) and the derivatives of the positions of the robots are numerically initialized as follows: $v_i = \varsigma_i = v_{di}$, $\dot{x}_i = v_{di}\cos\theta_i(0)$, $\dot{y}_i = v_{di}\sin\theta_i(0)$, where $v_{di} = 0.09$ [m/s] is the desired forward velocity, and $\theta_i(0)$ is the initial orientation of the robot.

<u>*Performance indicator*</u>

For performance evaluation, the following indicator is used:

$$P_T^{total} = \mu_{ind} P_T^{ind} + \mu_{form} P_T^{form} \quad \textbf{(27)}$$

where $0 \le \mu_{ind}, \mu_{form} \le 1$ are two scalar penalty weights, and P_T^{ind} and P_T^{form} are individual tracking and formation keeping performance indicators given as follows:

$$P_T^{ind} = \sum_{i=1}^{m} \sqrt{\frac{1}{l}\sum_{k=1}^{l}\left(e_{xi}^2(t_k) + e_{yi}^2(t_k) + \dot{e}_{xi}^2(t_k) + \dot{e}_{yi}^2(t_k)\right)} \quad \textbf{(28)}$$

$$P_T^{form} = \sum_{i=1}^{m}\sum_{j=i+1}^{m}\sqrt{\frac{1}{l}\sum_{k=1}^{l}\delta_{ij}^2(t_k)} \quad \textbf{(29)}$$

$$\delta_{ij}(t_k) = \Delta_{rij}(t_k) - \Delta_{ij}(t_k) \quad \textbf{(30)}$$

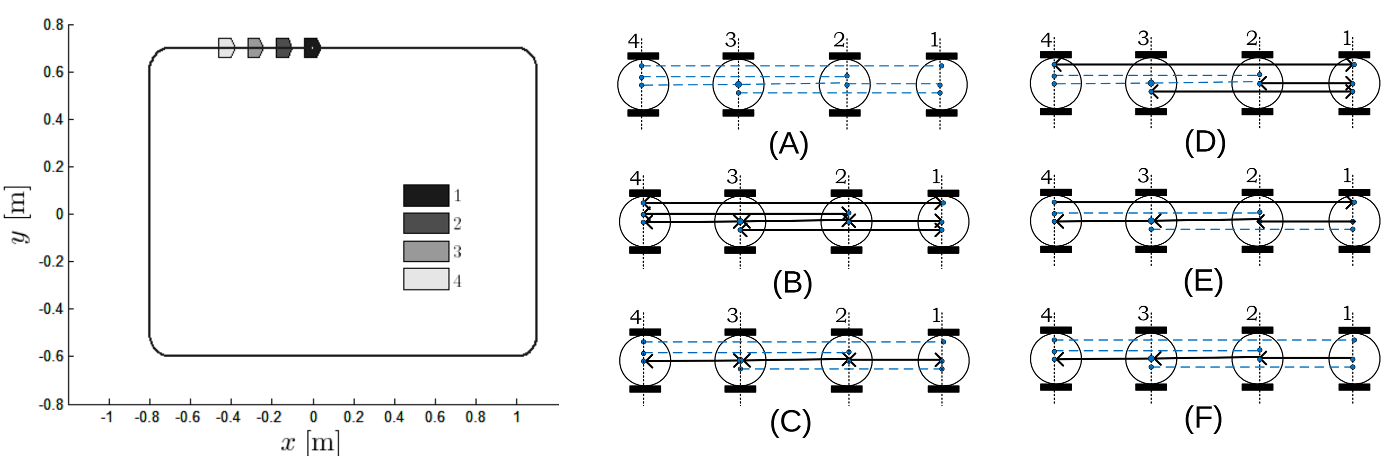
where m is the number or robots, t_k indicates the time instant where data is taken, l is the number of data in an experiment, Δ_{rij} and Δ_{ij} are the distance between the reference positions and the actual position of robot *i* and *j* respectively.

<u>*Experiment Scenarios*</u>

We deploy 4 robots in a platoon-like formation, of which the robots have to follow a rectangular like trajectories, see **Figure 3a**. Six different communication topologies are investigated as depicted in **Figure 3b**. The performance is analyzed in terms of different initial positions of the robots in the formation. Examples of videos of experiment can be seen at <u>www.youtube.com/adinandra98</u>.

<u>*Analysis*</u>

Figure 4 shows the movements of the robots in experiments using topology A and B. As seen in **Figure 4**-left, since topology A imposes no communication between the robots, there is no reaction from the robots that start from correct initial positions to compensate the formation keeping. On the other hand, when topology B is used this imposes full communication sharing between the robots. It can be observed that robot 1 and 2 tries to recover the formation first before following individual trajectories. This shows how the additional coupling to the control algorithm adds robustness in keeping the desired formation.



(a) **(b)**

Figure 3: (a) The reference trajectories and formation pattern for the unicycles. Robot 1 is at the right-most, robot 4 is at the left-most. Movement is in clockwise direction; (b) Communication topologies choices. The solid lines indicate if a connection exists. The open arrows indicate from which robots the information is obtained.

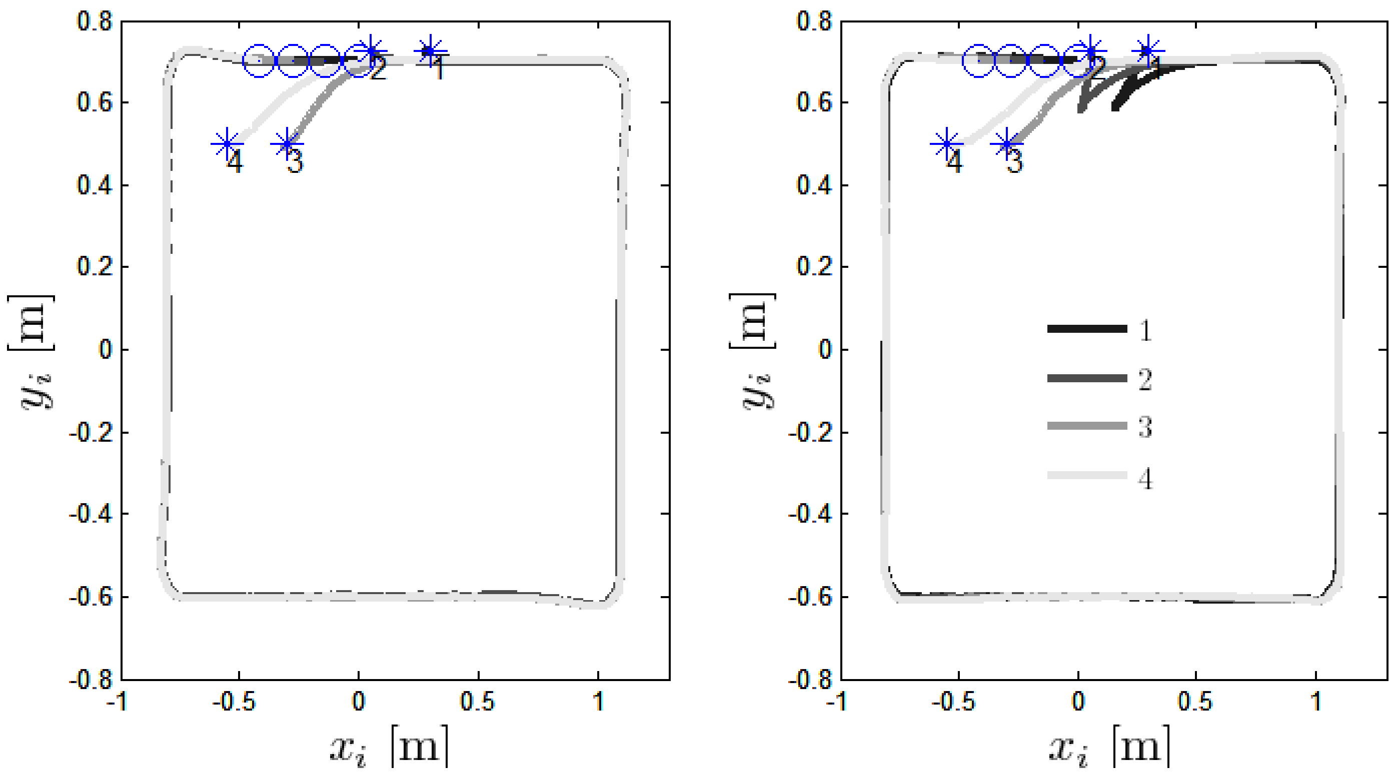


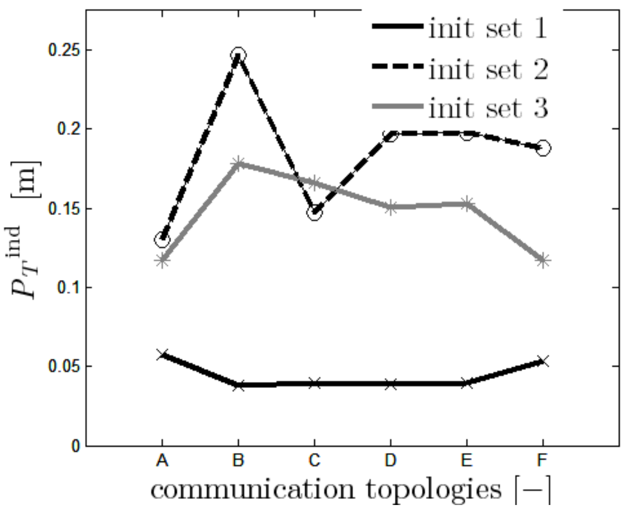
Figure 4 The complete movements of the robots using topology A (left) and B (right). The number indicates the starting points of the robots. The circle indicates the starting reference trajectories.

Furthermore, **Figure 5a**, **Figure 5b**, and **Figure 5c** shows the performance measure computed using (26)-(28). For (26) equal scalar weights are used. From **Figure 5a**, that shows P_T^{ind}, the best performance is obtained when topology A is used. The worst is obtained when topology B is used, regardless the choice of initial positions. Combine with the results from other topologies, it is suggested that if there are more connections between the robots, i.e. more information is being shared; the tracking performance tends to worsen.

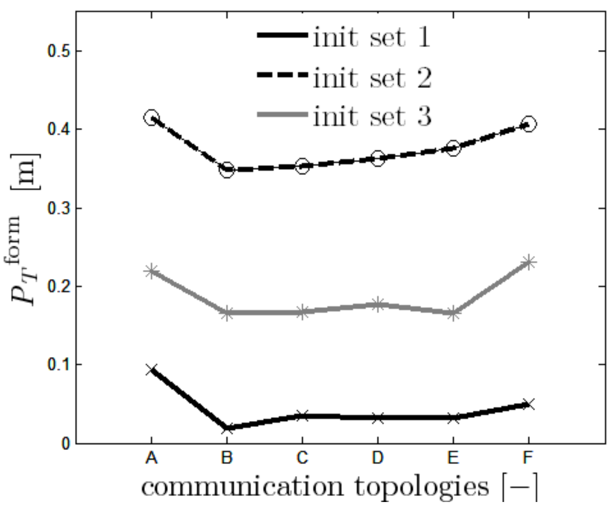
As for the formation keeping, **Figure 5b** shows that, regardless of the initial condition, the best P_T^{form} is obtained when topology B is implemented. On the other hand, the worst performance is obtained when topology A is used. These findings are opposed to the results in individual tracking performance. When all robots communicate, each of them knows what happens to other robots. When there is a perturbation, in this case the non-zero initial conditions, the group tries to preserve the formation first. As a consequence, the individual tracking errors become higher as observed in **Figure 5a**. The results in **Figure 5b** also suggest that formation keeping, regardless of the initial condition, will improve if there are more connections between the robots with the cost of requiring a higher communication bandwidth. If the number of robots is very large, this can be a disadvantage. From **Figure 5b**, one way to maintain a lower bandwidth while having a meaningful formation keeping performance is by choosing topology that allows connection with the neighboring robots, e.g. topology C or E.

Using equal weights, from **Figure 5c**, it can be observed that the total performance is dominated by the tracking performance. The best total performance for each initial condition set is obtained from different topologies. This suggests that, if tracking and formation keeping are equally weighted, communication topology has less influence on the total performance. The overall performance is mostly determined by the trajectory tracking performance. If either individual trajectory tracking or formation keeping is more important, the corresponding scalar weights need to be increased. In this way, more important indicator can suppress less important one.

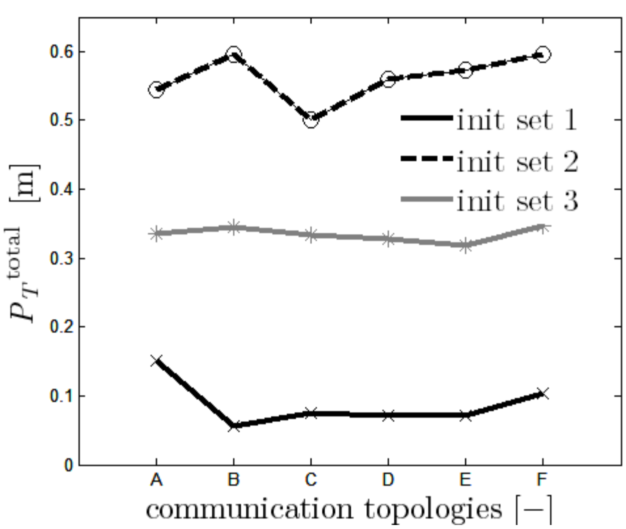
As additional validation, a different scenario as depicted in **Figure 6** is considered. In this scenario, the robots start from the same position as their individual references. During the process of completing the task, three perturbations are introduced at different times. The square signs in **Figure 6** indicate the positions of the robots when one of them is manually positioned to the point indicated by the star. The computations of trajectory tracking, formation keeping, and total performances are given in **Figure 7a**, **Figure 7b**, and **Figure 7c** respectively.



(a)



(b)



(c)

Figure 5 Performance measures from experiments using different topologies: (a) P_T^{ind}; (b) P_T^{form}; (c P_T^{total})

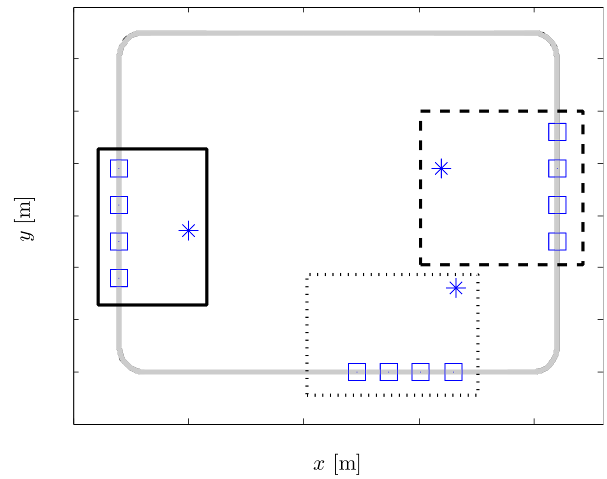


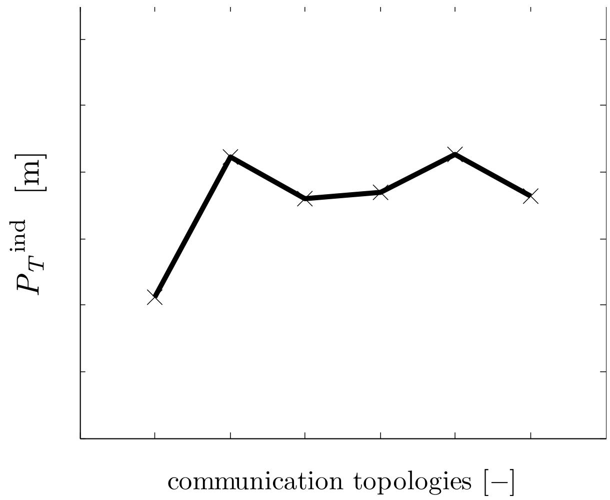
Figure 6 The reference trajectories and perturbations introduced during experiments; the rectangles indicate the positions of the robots when perturbations are introduced

As seen in **Figure 7a**, the best individual tracking performance is obtained when there is no communication between the robots, i.e. topology *A* is implemented, and vice versa. On the other hand, the best formation keeping is obtained when all robots communicate, i.e. topology *B* is implemented, and vice versa. Furthermore, **Figure 7c** indicates that using equal scalar weights, the best total performance comes from topology *A* although with a very small difference compared to topology, *B*, *C*, and *D*. These findings, similar to the results from the previous experimental scenarios, suggest that there is less need to share all information to achieve a good trajectory tracking and formation keeping performance.

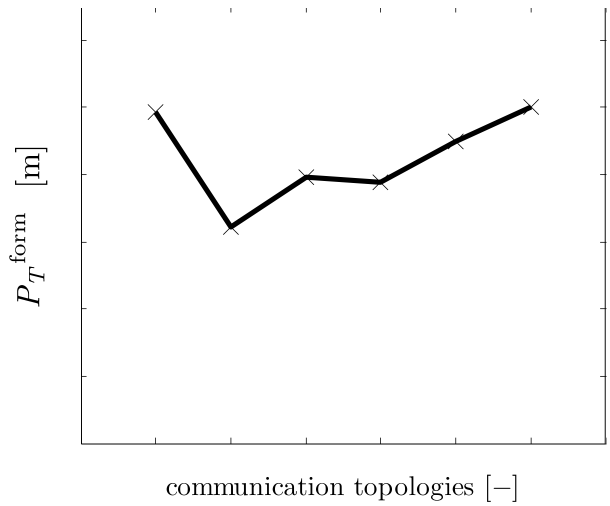
V. Conclusions and Future Works

This paper presents a simultaneous trajectory tracking and formation keeping control algorithm for a group of robots. The control algorithm is formulated by extending a trajectory tracking controller for a single robot developed using dynamic feedback linearization. The stability of the closed-loop system is analyzed using an existing theorem on interconnected systems. The algorithm has been successfully validated in real-time experiments.

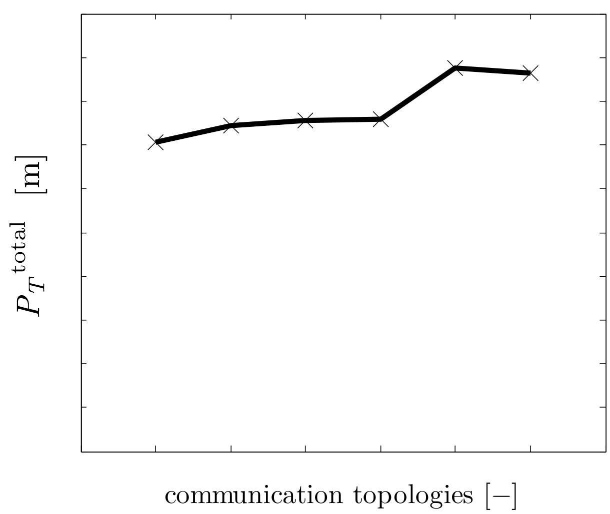
Using root-mean-square error-like indicators, the performance of the control algorithm is analyzed. The analysis of the experimental results suggests that the communication topology, which determines how the robots exchange information, has more influence on the performance compared to the initial conditions. The more information that is shared between the robots, the better formation keeping is expected, with the cost of a high communication bandwidth. The opposite result is obtained for the individual trajectory tracking performance. In this particular experimental case study, the best total performance with equal scalar weights result is obtained when there is no communication between the robots. This suggests that a good simultaneous trajectory tracking and formation keeping performance can be achieved with less information sharing.



(a) P_T^{ind}



(b) . P_T^{form}



(c) P_T^{total}

Figure 7 Performance indicators from the experiments using scenario shown in Figure 6.

ACKNOWLEDGMENT

This work has been carried out as part of the FALCON project under the responsibility of the Embedded Systems Institute with Vanderlande Industries as the carrying industrial partner. This project is partially supported by the Netherlands Ministry of Economic Affairs under the Embedded Systems Institute (BSIK03021) program.

The work has been done when the first author was affiliated with Eindhoven University of Technology.

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