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Traffic Flow Optimization by Weber-Fechner Law Applied Intelligent Vehicles

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Abstract—WeberFechner Law (WFL) that exists in the relationship between distance-headway (X) ($X = VT$, V: vehicle speed, T: time-headway) can be expressed like $X = X_0 e^{\beta V}$. If this can be embedded into vehicles as the driver assisting intelligence, it is expected to prevent rear-end collisions and to get high efficiency flows by easing congestions. Such benefits are theoretically mentioned from the development of the WFL. The essential core parameter is V_β which is defined from the WFL as *characteristic speed* which is also defined as *critical speed*, because when the flow speed $V = V_{av}$ (averaged speed) is equal to V_β $(V = V_\beta)$, the flow density becomes the maximum, which does not mean the highest flow efficiency, but it easily goes to congestion. If the speed V becomes equal to $2V_\beta$ ($V = 2V_\beta$), the maximum economic efficiency can be obtained.

Keywords—Flow optimization, Weber-Frechner Law, intelligent vehicle, characteristic speed, transportation volume.

I. Introduction

ALMOST all of research workers might have thought that the road traffic flow is one of the physical phenomena. However, the author of this paper wondered this idea because the traffic flow should be made by the drivers (human being). The author has found that the road traffic flow should obey the Weber-Fechner Law (WFL) in [1]-[4]. Owing to this finding, the theory of road traffic flow became a simple mathematical subject. The author has discussed the most of all related subjects concerning road traffic flow in his book in [5]. In this paper, the principle of the WFL applied road traffic flow will briefly be mentioned because it has been clearly mentioned in [6].

Among so many research works in the past, the author can pick a book up that mentions about the practically measured data analyses in [7], but the theoretical examinations have not been enough. We should not ignore the book written by Boris S. Kerner in [8] who has approached somewhat scientifically for analyzing a road traffic flow, but he has failed because of ignoring the human factor of driver. We can also pick up another book that mentions very theoretical approaches with mathematical ways for analyzing road traffic flow in [9], but it looks like a mathematical game.

If we view from the past theoretical research works on the road traffic flow, there are many estrangements between the theory and the practical situation, and almost all there are no endorsements of the measured data.

The articles to be mentioned here are the unifications of theory and measurements, which has been realized by the WFL. In this paper, we use the authorized unit SI (International System of Unit) instead of the traditionally used units in traffic flow like km/h (or kph) for speed.

II. Weber-Fechner Law (WFL) and Road Traffic Flow

Weber-Fechner Law (WFL)

The WFL is the law between physical stimuli (denoted by *W*) applied to human and its human feeling sensation level (denoted by *s*).

In 1840s, the works made by Ernst Heinrich Weber were formulated as a mathematical law (Weber's law in [1] [2]) that could be written as

$$k\frac{dW}{W} = ds \ (k\text{: constant}) \tag{1}$$

Equation (1) has been well interpreted by Michael Heidelberger in his book in [3] and briefly shown in [4].

The success to write the differential equation (1) of Weber's law let Fechner integrate it and got the Fechner's law written as

$$\mathbf{s = k\mathit{ln}(W) + C}, \tag{2}$$

where $\boldsymbol{ln}$ is the natural logarithm and C the integration constant, and C can be determined as $\mathbf{C = k\mathit{ln}(W_0)}$, where $\mathbf{W_0}$ is the value of when $\mathbf{s = 0}$. Thus finally we get

$$\mathbf{s = k\mathit{ln}(\frac{W}{W_0})} \tag{3}$$

or

$$\mathbf{ln(W_0) = \beta s + C}, \tag{4}$$

$$\mathbf{C = \mathit{ln}W_0 \text{ and } \beta = 1/k} \tag{5}$$

Expression of (5) is the well-known Weber-Fechner law. The common logarithm is available but we use the natural logarithm in this study because the traffic flow is thought to be the natural phenomena including a human (driver) factor, and also the phenomena related *e* (=2.71828..) are naturally essential which has been successfully applied in [5].

Instead of (4) or (5), the exponential formula of (6) is also available in the consideration of road traffic flow:

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$$W = W_0 e^{\beta s} \quad (6)$$

In our present study of the road traffic flow, as will be mentioned later, the exponential formula of (6) together with (4) or (5) will be used for the data analysis.

III. WFL AND ROAD TRAFFIC FLOW

Fundamental Equation of Road Traffic Flow

From the author's measurements of both speed V (m/s) of vehicles and time-headway T (s) (time interval of vehicle flow), we can get the distance-headway X (m) (see **Figure 6**) by multiplying V and T to get

$$X = VT. \quad (7)$$

A typical measured data of $V - T$ relationship is shown in **Figure 1** in which computer calculated approximate equation is shown in the form of

$$X = X_0 e^{\beta V} \ , \quad (8)$$

where X_0 and β are constants of which meanings will be mentioned later. The equation (8) can be defined as the WFL Equation for the road traffic flow.

The equation (8) has been well examined by the measurements made for all types of road conditions including intersection signals as we can see in the author's book in [5]. The essential measuring parameters are the vehicle speed V and the time-headway T, then X in (7).

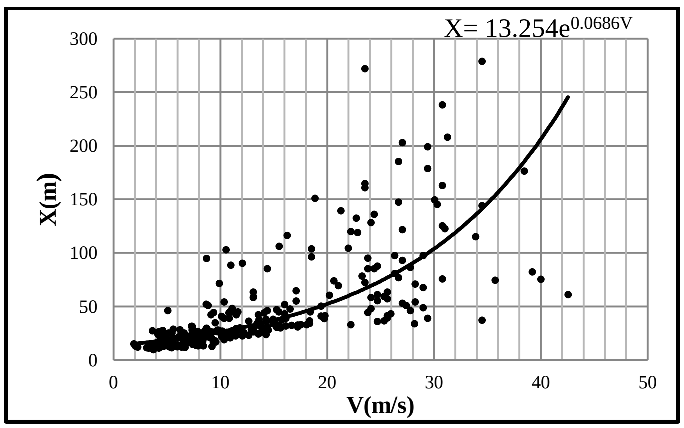

Figure 1 Measured $V - X$ plots and approximated equation (Tohoku Expressway in Sendai Japan)

The numerical example of (8) is shown in **Figure 1**, where in this case, the values of both X_0 and β were 13.254 and 0.0686, respectively. It is obvious that X_0 is the distance-headway when the speed $V = 0$, empirically it covers X_0=10~20m depending on the vehicle size.

The accuracy of the approximated equation shown in **Figure 1** should be examined. **Figure 2** shows the averaged data plots, $V_{av} - X_{av}$, on the log scale, which shows good linearity with the correlation factor of over 0.95, that shows very good approximation.

The original WFL equation for road traffic flow can be shown as a logarithmic equation like (9) from (4).

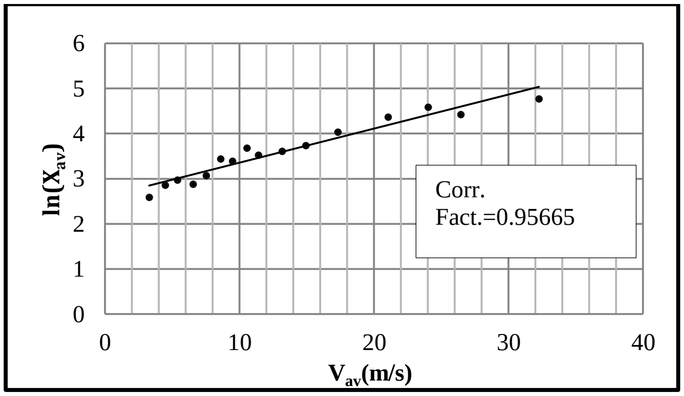

Figure 2 Averaged $V - ln(X)$ plots of the data of Fig.1

$$V = V_\beta \, ln\left(\frac{X}{X_0}\right) \ . \quad (9)$$

V_β in (9) is the specific speed that will be defined by (10). Theoretically, (9) becomes $V = 0$ when $X = X_0$.

Human Factors in Road Traffic Flow and WFL

Now, we have come to consider the relationship between the road traffic flow and the WFL.

In a long history of road traffic flow analyses, almost all researchers have thought that the vehicle flow phenomenon is a physical one. This standpoint was quite erroneous for the true understanding of road traffic flow, so that the scientific treatments have been erroneously left behind until today.

The road traffic flow is made by human drivers. Thus we should investigate it including the driver's behaviors.

The WFL is a law that characterizes the close relationship between stimuli to human (denoted by *W* in II.) and its sensing level by the human sensory system (denoted by *s* in II.)

For a vehicle driver, the distance L should be kept as a safety distance, where L is the distance from rear-end of the leading vehicle to the front-end of driver's one (see **Figure 6**). Our eyes perceive the distance L as stimuli in [3].

Maor Eli wrote in his book in [4] concerning WFL as follows: "Whereas physical stimuli are objective quantities that can be precisely measured, the human response to them is a subjective matter". This sentence can be applied to the present situation of vehicle drivers. That is to say, in case of vehicle driver, the physical quantity L (see **Figure 6**) is the objective quantity that the driver's eyes can recognize it by looking it as the physical stimuli. On the contrary to the recognition of distance L, the sensation of speed is the subjective matter so long as we are onboard.

Summarizing the above, we can conclude as follow:

1. Information recognized by eyes is a stimuli to the drivers
2. Driver's recognition of vehicle speed is a subjective matter

From the above conclusions, we are convinced of that there exists the Weber-Fechner Law (WFL) in road traffic flows, as long as the vehicle drivers are concerned,

IV. CHARACTERISTIC SPEED V_β AND TRAFFIC FLOW

Definition of V_β

V_β is the key parameter in management of road traffic flow that will be mentioned hereafter. We define V_β form the WFL as follows.

In the WFL in (8), the specific speed is defined, that is the speed V in case where

$$V = \frac{1}{\beta} = V_\beta. \tag{10}$$

It comes from $V\beta = 1$ in (8). At this condition, X should be

$$X = eX_0 \approx 2.73X_0 \, . \tag{11}$$

As mentioned in the previous Section II, $X_0 \approx 10 \sim 20m$, then at this condition, $X \approx 30 \sim 60$m.

Here we define the specific speed V_β as the *characteristic speed*, because the speed V_β is the inherent constant of the specific position of road, and its value should be different from each other at position to position along even on the same road due to slopes, curves, impediments, and etc. As will be shown in **Figure 3**, V_β is also called *critical speed* because it is the border speed between non-congestion and congestion flow.

Value Estimation of V_β

V_β is the inverse value of β as shown in (10). Our measurements for several road traffic flows have revealed that the parameter β depends on the easiness of driving, and the easier road for driving has a smaller value of β, which means the bigger value of V_β. It is natural that good road has a bigger V_β.

Table 1 show the several examples of the values of V_β which were calculated by (12). The formula of (12) was developed by the author in [5].

$$V_\beta = \frac{V_{tav}}{1+\ln(T_{tav}/T_{<2,3})} \tag{12}$$

The suffices 'tav' in (12) means 'total average' and '<2, 3' means 'less than 2 sec, or 3sec'. $2V_\beta$ in **Table 1** is the economically optimum speed that has been introduced in [5] and [6], which will be briefly mentioned:

● The traffic density should be the maximum at the condition of $V = V_\beta$ (T_{min} in (14)), which may be thought the highest efficiency from the economical viewpoint (see Section V). But this idea is not correct. The author verified the maximum economical traffic efficiency should be in the case of $V = 2V_\beta$ in [6].

V_β and Flow

-In case of $V = V_\beta$ -

The flow at $V = V_\beta$ is the maximum flow rate that means the time-headway T becomes the minimum (T_{min}), which can be derived from the fundamental WFL Equation (8). Since X = VT, T can be written like

$$T = \frac{X_0}{V} e^{\beta V} \tag{13}$$

TABLE 1 V_β'S OF SEVERAL ROADS

Road \ Data		$T_{<2,3}$ (s)	T_{tav} (s)	V_{tav} (m/s)	V_β ($2V_\beta$) (m/s)	Note
Tohoku Exp. (Sendai)	Hitokita Up	1.62	3.66	22.7	12.5 (25)	1)
	Down	1.52	4.66	28.7	13.5 (27)	2)
	Oritate Up	1.46	3.56	31.7	16.7 (33.4)	3)
	Down	1.19	2.56	34.2	19.3 (38.6)	4)
Nara (National No.25)		2.08	6.55	12.9	6.21 (12.4)	5)
Kita Sendai		1.98	7.16	11.4	5.08 (10.16)	6)
Sendai Higashi 2	To North	1.97	4.66	8.71	4.80 (9.6)	7)
	To South	1.85	4.57	11.7	7.29 (14.58)	8)
Sapporo		1.86	3.73	7.65	5.06 (10.16)	9)
Nakayama		2.16	8.41	9.65	4.01 (8.02)	10)

1) to Tokyo (Uphill-slope) * $T_{<2}$ for 1)~4)
2) to Sendai (Downhill-slope)
3) to Aomori (Uphill-slope)
4) to Tokyo (Downhill-slope)
5) 7 days all data averaged
6) Center road from north to center in Sendai City
7) Sendai Higashi 2 Bancho Dori (Viewing north from SS30Building)
8) Opposite (south) side of 7)
9) Inside flow in Intersection (observed from Sheraton Hotel)
10)Residential area main road

This function means that T becomes infinity in case where V goes to both zero and infinity. At the condition of $V = V_\beta$, T becomes the minimum ($T = T_{min}$). The actually measured data is shown in **Figure 3**.

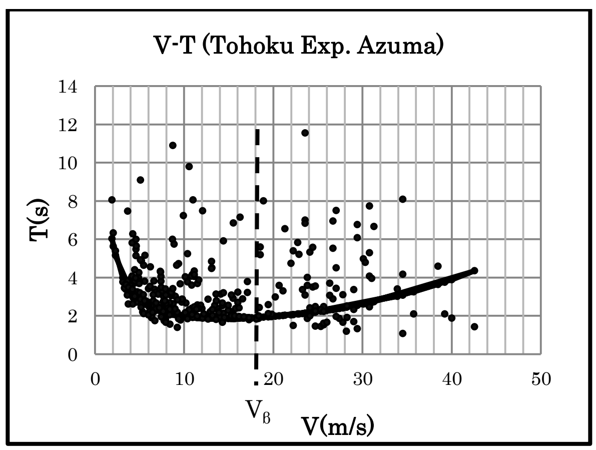

Figure 3 Measured V − T data and V_β

The data in **Figure 3** are the same with those in **Figure 1**. The V − T function in (13) has a concave characteristics and it has the minimum value at the specific value of V. From the simple mathematical procedure, we can get the specific value of V, that is V_β, of which value in the case of Tohoku Expressway at Oritate, Sendai is about 18 m/s ($V_\beta \approx 18$ m/s ≈ 65 km/h) as shown in **Table 1**. Then the minimum $T(T_{min})$ becomes

$$T_{min} = \frac{eX_0}{V_\beta} = \frac{30 \sim 60}{18} = 1.7 \sim 3.3s. \tag{14}$$

The flow speed at the condition of (14) becomes $V = V_\beta$ as shown in **Figure 3**, and the flow can be depicted like **Figure 4**, where the distance-headway is eX_0 in average. The average distance-headway eX_0 is about 30~60m as was shown in (11).

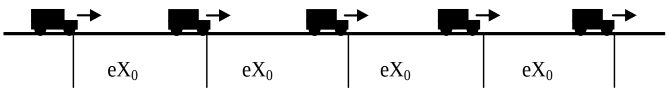

Figure 4 Car flow at the speed of $V = V_\beta$

V_β and Flow

-$V < V_\beta$ (Congested Flow) -

As shown in **Figure 3**, T increases in the region of $V < V_\beta$, and the flow should go to congestion. And occasionally $V = 0$ should occur. This situation is the ultimate state of congestion where all vehicles cannot move and long line of queue appears. The typical case of this situation we can see is the queue made at red signal at an intersection. Just like the case of red signal, the stationery vehicle line appears that can be shown like that shown in **Figure 5**. At this condition, the distance-headways become shortest to be $X = X_0$ ($\approx 10 \sim 20m$) in average.

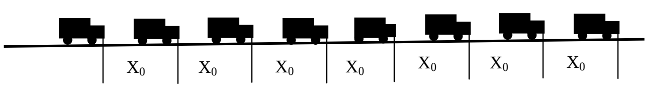

Figure 5 Ultimate state of flow in case of $V < V_\beta$

Variety of Value of V_β as Characteristic Speed

From mentioned above, V_β is the *critical speed* in a road traffic flow. Here, we should be noticed that V_β has a different value from place to place along the road in [5] and [6]. Thus we should admit that V_β is the *characteristic speed* which can be partially applied along the road. The values should be estimated from the calculation of (12) by applying the data measured at a position to another position along the road which can include intersections, then V_β should have a different value at a different position along the road. In the practical cases, we should choose the smallest value of V_β along some span of the road.

V. TRAFFIC FLOW BY WFL APPLIED INTELLIGENT VEHICLES

Comment on Driverless Car

Here we have come to discuss on the future version of traffic flow with the intelligent vehicles. The driverless car is seemed to be a main subject and has been tested in [10]. The technological advancement to make a driverless car should be admired, but we should notice if any fundamental theories have been applied. The needs of driverless car will increase in future as one of the robots to work at any place where human cannot approach. In that case, the full technologies of intelligence should be necessitated to be embedded into vehicles. And the author also agrees the counter opinion against driverless car mentioned in [11] which mentions the driverless car should be limited because many people enjoy the car driving.

Strategy to Avoid Congestion

From the above description concerning the role of V_β, the strategy to avoid a congestion is very simple. As mentioned in **V_β and Flow**, the vehicle speed should not be smaller than V_β ($V > V_\beta$). This speed control algorithm can easily be achieved if we embed the WHL into every vehicle, but in order to persuade this algorithm, the near future traffic forecast is necessary. Although the near future traffic flow forecasting is not perfectly available today, the author has presented the effective idea in [6].

We should note that in order to persuade to avoid congestion, the information network system between the flow managing center and vehicles should be installed, as it has been proposed in [6]. The necessitated information is the recommended speed basedupon $2V_\beta$. The adaptive sign-board has been proposed in [6].

WFL Applied Braking System for Avoiding Collision

– Principle –

When the leading car suddenly decelerates or due to carelessness of the driver of following vehicle, the rear-end collision may happen accidentally. We need some counter method to prevent it. This accident can be decreased if we adopt the WFL embedded braking system into the vehicles.

The WFL in vehicle flow has been formulized like those shown in (8) or (9).

The accident of rear-end collision is very often, many of which can occur due to the following vehicle driver's carelessness. The first priority of intelligence should be its prevention system.

It looks easy to embed the WFL into vehicle braking system, because its algorithm is not so complicated. But we should not forget that the intelligent braking system should be auxiliary to assist the driver.

The brief algorithm for intelligent braking system is as follows:

1) Sensor of distance L (see **Figure 6**) should be installed
2) Calculation of speed (V) by the formula (9) (replace X to L) should begin with respect to the sensor output of L when it becomes close to some threshold (e.g. 150 m for high speed way, 100m for ordinary road)
3) Braking system should be operated by the calculated result to let the speed be equal to that of leading vehicle.
4) When the leading vehicle stops, the following one should stop with the safety distance X_0 from the leading vehicle.

—Model —

The driver's eyes can recognize the distance L shown in **Figure 6**. The sensor should also detect it.

Distance L between vehicles is

$$L = X - \Delta L. \qquad (15)$$

The automatic braking system should be active when L becomes 100~150 m.

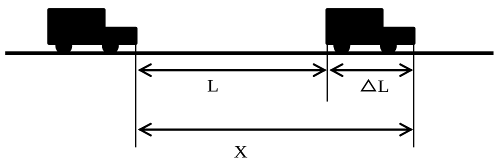

Figure 6 Distance L between two vehicles

— X-V Characteristic Curve—

X in (15) has been shown in (8) in case where both vehicles are moving with the same speed of V . But when we consider the case of collision, the speed of both vehicles should be quite different; the case can exist where the leading vehicle is not moving. Nevertheless, we can discuss the collision avoiding system by the WFL.

Equation (9) is another formula of WFL. By substituting $X-\Delta L$ instead of X, (9) can be written as

$$V = V_\beta ln(\frac{X-\Delta L}{X_0}) \quad (16)$$

The numerical calculation is shown when $L\ (= X-\Delta L)$ becomes X_0 ($X-\Delta L \rightarrow X_0$)then V becomes zero ($V = 0$). The result is shown in **Figure 7**, where we assumed here $X_0 = 20\ m$.

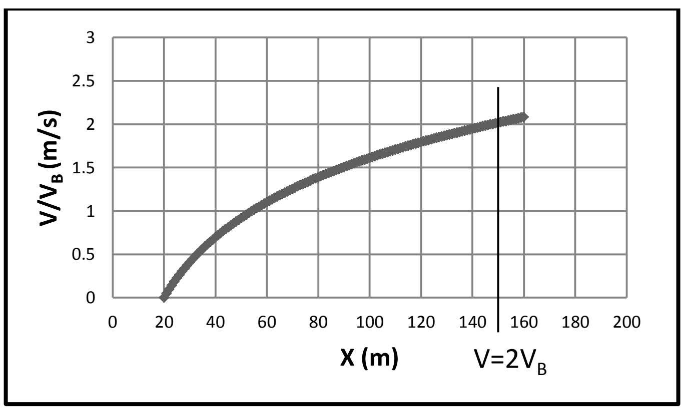

Figure 7 Calculated results of (16)

The numerically calculated curve shown in **Figure 7** can be installed into all braking system of vehicles, although the value of V_β should be determined, but it should not be difficult to empirically choose some appropriate value.

VI. Evaluation of Traffic Flow from Economy

Definition of Flow Rate

The traditional definition of the flow rate q is

$$q = \frac{1}{T}. \quad (17)$$

The unit of q is [s^{-1}]. Thus (17) means the number of vehicles per 1 sec. If two vehicles passed at a position of road within 10 sec in average, q becomes 0.2/s, which means one min (60 s) flow rate becomes 12 vehicles per min.

At the specific speed of $V = V_\beta$, the flow rate (17) becomes the maximum ($q = q_{max}$). The averaged flow of this condition has been shown in Fig.4, where the average distance-headway becomes eX_0 and flow density at this condition becomes the maximum because T becomes the smallest like that shown in (14). The flow in the case of $T = T_{min}$ means the maximum flow rate. But it cannot be the most efficient flow from the economical viewpoint that will be mentioned in the following (Refer to ● in Section III).

Difference of Viewings Flow from T and V

The flow rate q was defined by (17). The traditional flow rate has been shown by the number of vehicles passed the specific point during a specific period, for example, 5 min (300 sec), here we write the number of vehicles passed is N_{300}. The average time-headway T should be $300/N_{300}(s)$, then the flow rate defined by (17) will be

$$\frac{1}{T} = \frac{N_{300}}{300}. \quad (18)$$

We shall mention the data like (18) has almost no meanings that will be shown here.

Figure 8 shows the actual measured data of T on the expressway at a time of both congestion and free flow. The flow rate is the inverse values of **Figure 8** of which abscissa shows the time in second. **Figure 9** shows the vehicle speed corresponding to **Figure 8**.

If we look at the pattern of T vs. time t, we admit that almost no information can be derived from this pattern. This means the flow rate q defined by (17) and practical expression like (18) has almost no meaning.

However, if we look at the measured speed V shown in **Figure 9**, we can be convinced of the flow activity from the variation of speed.

We should admit that the time-headway T and the speed V are almost independent parameters each other, although it looks that there exist some correlations between T and V in the case when the flow becomes heavily congested as we can see the pattern of $t-T$ in 220~500sec in **Figure 8**. But in case of the normal flow, they are independent. From the above facts, we should note that when we consider the traffic flow, the measurement of T alone gives us almost no meaning, but as shown in (7) and (8), T is the core parameter along with V for characterizing the traffic flow.

On the contrary to T, the quite different pattern of $t-V$ is shown in **Figure 9**. We should admit that the variation of vehicle speed V with respect to time t can show the traffic flow activity.

Definition of Transportation Volume Q

As mentioned above, the flow q itself cannot be used as the evaluation parameter of flow. Here, we define a new parameter Q as *transportation volume*:

$$Q = qV = \frac{V}{T} \quad (19)$$

The dimension of Q is m/s^2 that means the acceleration in case of mass movement.

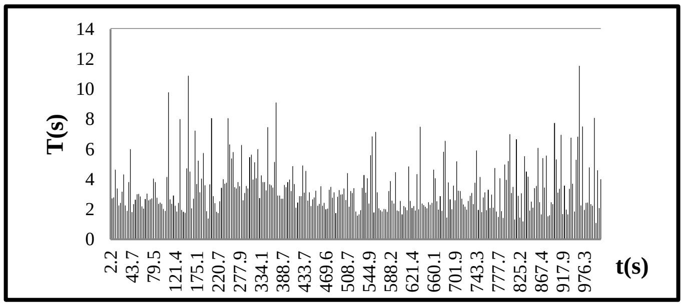

Figure 8 Measured time-headway T

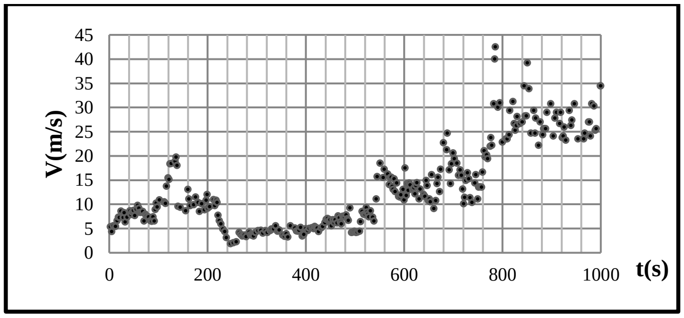

Figure 9 Vehicle speed corresponding to Figure 8

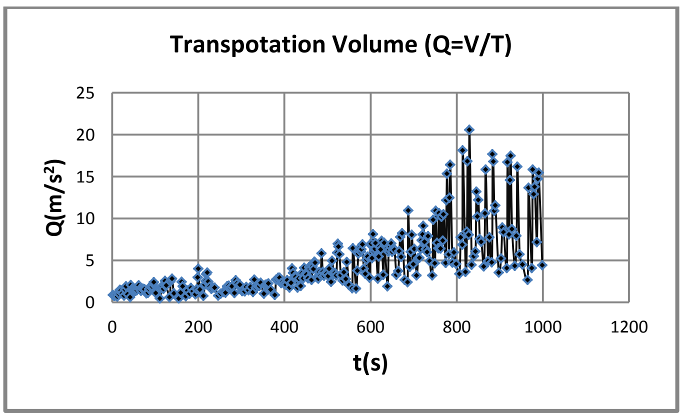

Figure 10 Transportation Volume Q in actual flow

Figure 10 shows the transportation volume Q of the flow shown in **Figure 8** for T and **Figure 9** for V. We are able to evaluate the transportation efficiency by Q.

The final evaluation of the road traffic flow can be made by Q. Very interestingly the maximum Q ($=Q_{max}$) can be obtained at the speed of twice of V_β ($V=2V_\beta$) in [6].

We have reached to conclude that the road traffic flow can be evaluated from the economical viewpoint. The speed of $2V_\beta$ can be used to indicate to drivers by the adaptive sign board as the recommendation speed, in [6].

VII. Conclusion

We have seen in this paper that the road traffic flow phenomena are governed by WHL (Weber-Fechner Law). Owing to this fact, we could have the theoretical advance in the field of road traffic flow which was once long time being thought as a complicate phenomenon. The theory has been supported by the actual data of measured road traffic flow. Moreover, some new proposals could be done for evaluation of flow such that the traditional way of evaluation of flow by counting the number of vehicles passing at any fixed positions on road has no effective or practical meanings. The essential parameters to be measured should be both the timing (T) and speed (V). The specific speed V_β was derived and shown that it is a core parameter in almost all needs for optimization and intelligence for the road traffic flow. The final proposal was the economical evaluation method by newly defined Transportation Volume Q. The maximization of Q should be the final target for road traffic flow optimization.

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