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Cable Angle Feedback Control for Helicopter Slung Load System Using Delayed Feedback

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***Abstract*—**In recent years, a large number of small-size helicopters are used for some industrial works. In this study, we focus on vibration control for a small-size helicopter with a slung load system. We employed delayed feedback control to suppress vibration of the load by measuring its cable angle. We verified the effectiveness of the vibration control method by real flight tests. As for the controller design, it is difficult to determine the delayed feedback parameters because delayed feedback control is one of the nonlinear control methods. We demonstrate a way to design the controller considering the rotational and the translational dynamics and the system parameters required for the design were identified by the frequency response test. To simplify the subject, we built a simple planar model like a double pendulum and the delayed feedback controller was designed by root locus of the system. The optimum delayed feedback control parameters derived theoretically are nearly identical to the parameter determined by trial-and-error in the experiment.

***Keywords*—**small-size helicopter, delayed feedback control, vibration control, system identification.

I. Introduction

Vibration control methods for a small-size helicopter with a slung load system are important for achieving a secure and convenient transport. A small-size helicopter has several characteristics. The motion is fast and sensitive in comparison with normal large-scale helicopters and tends to be unstable to disturbances because mass of the fuselage is relatively light. To achieve a secure transport, the control system needs to suppress the vibration of the slung load to minimize extraneous disturbance factors. In this paper, we focus on the vibration control method for a small-size helicopter with slung load system during hovering, and discuss the strategy to design the controller.

A. Related work

During the 1960s and 1970s some papers dealing with heavy lift helicopters were published. To our knowledge, the first theoretical report of the system was described by Lucassen and Sterk [1]. The early investigation reported by Dukes demonstrated the stability analysis of the system during hovering and slow speed flight based on a simple model which has three degrees of freedom [2-3]. Some studies investigated high speed flights considering the aerodynamics acting on the load. Poll and Cromack demonstrated that the helicopter slung load system in high speed flight tends to be stable with long cable and heavy load [4]. Cicolani *et al.* reported the air flow dynamics acting on a slung load [5]. As a study dealing with the identification for the helicopter slung load system, Sahai *et al.* implemented the frequency response test and analyzed the data based on the flight software (CIFER) [6].

To suppress the vibration of the slung load, a number of studies have been made to add the damping effect to the slung load by feedback control. Ivler *et al.* investigated the optimum feedback parameter of cable angle/rate feedback to actualize the vibration control of the load and the attitude control of a helicopter simultaneously by solving a fundamental trade-off problem between load damping and piloted handling qualities [7]. Furthermore, they demonstrated the switching control between the cable angle feedback and the simple fuselage feedback to improve piloted handling qualities [8]. Oktay and Sultan built a model composed of 32 equations and considered airflow dynamics. They applied modern control theory to the system for energy efficiency and safety flight [9].

Up till now relatively few studies have been reported on systems carrying a load by using small-size helicopters. Bernard *et al*. demonstrated a transportation method to carry a heavy load by using multiple small-size helicopters [10]. The study has focused on tension control of the cables connecting the load to several helicopters, however, it does not mention about the vibration control of the load. The vibration control method was reported by Bissgard *et al.* who applied delayed feedback control to the system [11]. Position and velocity of the load were estimated by using unscented Kalman filter from the data measured by a vision sensor and an inertial measurement unit (IMU) [12]. Additionally, they made an analytical model for the constrained system by applying Udwadia-Kalaba method [13]. In a cruising situation, a few studies tried to apply the input shaping technique to achieve the small vibration transport of the slung load [14-15].

B. Summary of this study

The purpose of this study is to develop a simple and practical vibration control method for a helicopter with slung load system based on the delayed feedback control, and to establish a way to design the controller considering the rotational and translational dynamics of the helicopter. Our control strategy divides the control system into the attitude control of the helicopter (inner-loop) and the vibration control of the slung load (outer-loop). The attitude of the small-size helicopter is generally controlled by the classic control system theory such as the proportional-derivative feedback control in terms of roll and pitch dynamics. In this paper, the position of the helicopter and the vibration of the load is controlled by the reference angles of

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the attitude control. For the vibration damping, the reference angles are determined only by lateral and longitudinal angles of the cable connected to the load based on the delayed feedback control theory, so that the method does not need the information of angular velocity of the cable. The method enables us to achieving vibration control by using a low-priced instrument measuring the cable angle.

The problem of the present control method is that it is difficult to design the controller due to its nonlinear characteristic. We built a simple planar model concentrating on longitudinal or lateral dynamics to simplify the controller design problem. Since the accurate system parameters including the dead time are required for the controller design, we implemented frequency response tests to identify them in terms of the rotational and the translational dynamics, respectively. Consequently, we designed the delayed feedback controller by the root locus.

In past studies dealing with small-size helicopter slung load systems, Bisgaard *et al*. systematically summarized the vibration control method, the state estimation, and the controller design [11-14] as mentioned above. Compared to their studies, we tried to add the vibration damping by using a simple low-cost angle measuring instrument. The present control method consists of just a cable angle feedback control, so that it does not need any estimators and filters for the slung load. In addition, we designed the delayed feedback controller considering not only the rotational dynamics, but also the translational dynamics. We tried to derive the delayed feedback parameters to achieve the attitude control of the helicopter and the vibration control of the load simultaneously.

To verify the effectiveness of the present vibration control method, we implemented flight tests with a small-size helicopter with slung load system. Then, the delayed feedback control parameters were determined by trial-and-error as a first step. After the flight test, we tried to theoretically design the delayed feedback controller. The validation of the design strategy was confirmed by comparing the parameters of the optimum control with the parameters derived from trial-and-error in the flight tests.

II. Vibration Control Method

A. *Coordinate of control subject*

The control subject is composed of a helicopter and a slung load connected by a cable. We regard the helicopter as a rigid body and the slung load as a point mass. The position of the load is expressed by the cable angles on lateral (x_b - z_b) and longitudinal (y_b - x_b) plane of the helicopter. The schematic is shown in **Figure 1**. The attitude of the helicopter is expressed by ϕ_r, ϕ_p and ϕ_y, which correspond to roll, pitch and yaw angles, respectively. The cable angles of the slung load are defined as θ_r and θ_p.

B. *Vibration control method*

This paper assumes that the classic control system theory is applied to the subject because sensing system of small-size helicopters is used to be composed of popular prices sensor and the control theory enables us to adjust control parameters easily in the system. The control system is generally regarded as four

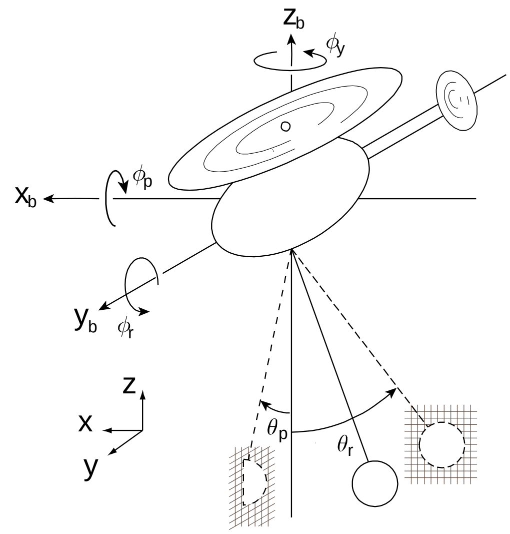


Figure 1 Schematic view of a helicopter with a slung load system

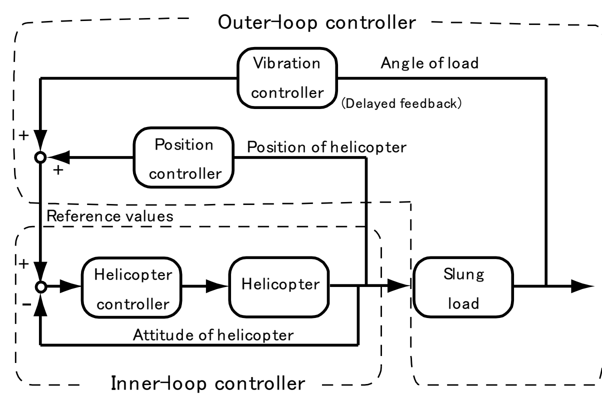


Figure 2 Block diagram of the presented control system

single input and single output (SISO) systems of roll, pitch, yaw and vertical dynamics. Cyclic pitch of the flapping angle generates torque for the rotational dynamics of the helicopter. When the attitude of helicopter is controlled by proportional and derivative feedback control, the reference angles are added to the system as the outer-loop. Therefore each control input for roll and pitch dynamics is given by

$$\begin{aligned} u_r &= -k_{dr}\dot{\phi}_r - k_{pr}(\phi_r - \bar{\phi}_r) \qquad \text{(Roll)} \\ u_p &= -k_{dp}\dot{\phi}_p - k_{pp}(\phi_p - \bar{\phi}_p) \qquad \text{(Pitch)} \end{aligned} \tag{1}$$

where u_r, u_p are the control input for cyclic pitch, and ϕ_r, ϕ_p are angles of the helicopter. Subscripts r and p mean roll and pitch respectively. Feedback parameters k_p, k_d were determined by trial-and-error in flight tests without slung load. In the system, $\bar{\phi}_r$, $\bar{\phi}_p$ are the reference angles which are composed of the reference angle for the position control of the helicopter and that for the vibration control of the slung load as follows:

$$\bar{\phi}_r = \bar{\phi}_x + \bar{\phi}_{\theta r}, \qquad \bar{\phi}_p = \bar{\phi}_y + \bar{\phi}_{\theta p} \tag{2}$$

where $\bar{\phi}_x$, $\bar{\phi}_y$ are the reference angles for the position control and $\bar{\phi}_{\theta r}$, $\bar{\phi}_{\theta p}$ are that for the vibration control.

The reference inputs for the position control are given by proportional and derivative feedback control as follows:

$$\begin{aligned}\bar{\phi}_x &= -K_{px}\{x(t-\tau_g)-\bar{x}\} - K_{dx}\dot{x}(t-\tau_g)\\ \bar{\phi}_y &= -K_{py}\{y(t-\tau_g)-\bar{y}\} - K_{dy}\dot{y}(t-\tau_g),\end{aligned} \quad (3)$$

where K_{px}, K_{dx}, K_{py} and K_{dy} are the feedback gains of the position control and $\bar{x}, \bar{y}$ are the references for the position control. Dead time τ_g means the time lag of GPS measurement, which should be more than 0.5 sec in general.

We employed the delayed feedback control for the vibration control of the load. The reason is that it is hard to estimate state variables of the whole system containing the load without IMU or vision sensor because the system is a constrained system. The delayed feedback control is simple and effective to suppress the vibration by cable angle feedback. In this study, two angles of the slung load can be measured by a simple angle measuring device with two potentiometers. Consequently, the reference angles are given by

$$\bar{\phi}_{\theta r} = G_{dr}\theta_r(t-\tau_{dr}), \quad \bar{\phi}_{\theta p} = G_{dp}\theta_p(t-\tau_{dp}), \quad (4)$$

where τ_{dr}, τ_{dp} are the fixed time intervals and G_{dr}, G_{dp} are the feedback gains. The control enables us to control the vibration the slung load and the position of the helicopter simultaneously.

Over all, a block diagram of the control for roll and pitch dynamics is shown in **Figure 2**. In inner-loop, the attitude of the helicopter is controlled directly by proportional and derivative feedback control. In outer-loop, the position control of the helicopter and the vibration control of the load are superposed on the attitude control through the reference angles. While control design parameters of the attitude control and the position control of the helicopter were determined by trial-and-error testing in advance, those of the vibration control (τ_{dr}, τ_{dp}, G_{dr} and G_{dp}) should be determined theoretically because the flight test with slung load is accompanied with some risk.

III. EXPERIMENT FOR VERIFICATION

A. Helicopter for flight test

Align T-Rex600CF shown in **Figure 3** was used for our flight tests. The helicopter is attached with a heading reference system (Microstrain 3DM-GX3-25), GPS (Garmin GPS18x-5Hz), and a pressure sensor (VTI Technologies SCP1000). As mentioned above, the helicopter is controlled by the single-input-single-output proportional derivative feedback control system in terms of roll, pitch, yaw, and heave motion. Cycle time of the system is 50 ms [16].

For the flight test, a load and a device for measuring angles of the slung load shown in **Figure 4** are added to the helicopter. The cable angles can be measured by the device with two potentiometers. The helicopter with measuring apparatuses has a weight of 6.06 kg and the load has a weight of 0.73 kg. The load is connected to the helicopter by a cable whose length is adjustable from 0.1 to 4.6 m. We fixed the cable length to 3.0 m in this study.

B. Experimental result

We verified the effectiveness of the present control method experimentally. Feedback parameters are set as follows: $k_{dr} = k_{dp} = 17.0$, $k_{pr} = k_{pp} = 1031$, $K_{dr} = K_{dp} = 0.0105$, $K_{pr} = K_{pp} = 0.0785$, $G_{dr} = G_{dp} = 0.022$, $\tau_{dr} = 0.6$, $\tau_{dp} = 0.8$. These parameters were determined experimentally by trial-and-error when the cable length was fixed at 3.0 m and they are near-optimum parameters in our equipment. To secure the safety of experiments, we assigned a limit to the amplitude of the reference angles $\bar{\phi}_r$, $\bar{\phi}_p$ within 20 degrees. This test was implemented during hovering. The first step in the test is to intentionally cause vibration of the load. Afterwards the vibration control was activated.

Figure 3 Flight test of a helicopter with a slung load system

Figure 4 Angle measuring device with two potentiometer

The result is shown in **Figure 5**. The vibration control was activated at about 4 sec in this test. The result demonstrates that the amplitude of the slung load was gradually attenuated with keeping the helicopter hover. The nature of the present control method is to converge slowly but surely. Although the details of the experimental results in increasing G_d are omitted here for brevity, the vibration of the slung load was suppressed faster and the helicopter did not keep the hovering position when G_d were set relatively high.

This method has a strong point that it does not need any filters or state estimators for the dynamics of the load. However, one limitation of this method is that it is difficult to design the delayed feedback controller. The rest of the paper is devoted to how to design the delayed feedback control parameters.

IV. Modelling and System Identification

A. Definition of the helicopter model

To design the delayed feedback controller, we defined a helicopter model and identified the system parameters. In general, a helicopter can fly without knowing the system parameters because the feedback parameters are tuned by a user or an auto-tuned system. However, the system parameters are required when we try to design the delayed feedback controller by using an analytical model. First of all, we defined a helicopter model by referring to Yamaha R-50 modelling reported by Mettler *et al*. [17].

The equations of motion of the translational and the rotational dynamics in roll/lateral and in pitch/longitudinal are given by

$$\ddot{x} + c_x \dot{x} = \gamma_x \phi_r, \qquad \ddot{y} + c_y \dot{y} = \gamma_y \phi_p \tag{5}$$

$$\ddot{\phi}_r = G_a a, \qquad \ddot{\phi}_p = G_b b \tag{6}$$

Where subscripts a and b mean the lateral and longitudinal rotor flap angles and c_x, c_y, γ_x, γ_y, G_a and G_b are the system parameters. The lateral and longitudinal flapping dynamics of the main rotor are given by

$$\begin{aligned} T_a \dot{a} &= -a - T_a \dot{\phi}_r + \alpha u_r (t - \tau_{lr}) \\ T_b \dot{b} &= -b - T_b \dot{\phi}_p + \beta u_p (t - \tau_{lp}) \end{aligned} \tag{7}$$

where T_a, T_b, α and β are the system parameters. The control inputs u_r, u_p have time lags τ_{lr}, τ_{lp} due to the dead time of the DC servo motors.

From Eqs. (6) and (7), we obtain the transfer functions between the control inputs and the roll and pitch angles of the helicopter as follows:

$$\frac{\Phi_r}{U_r} = \frac{G_a \alpha e^{-\tau_{lr} s}}{T_a s^3 + s^2 + G_a T_a s}, \quad \frac{\Phi_p}{U_p} = \frac{G_b \beta e^{-\tau_{lp} s}}{T_b s^3 + s^2 + G_b T_b s}. \tag{8}$$

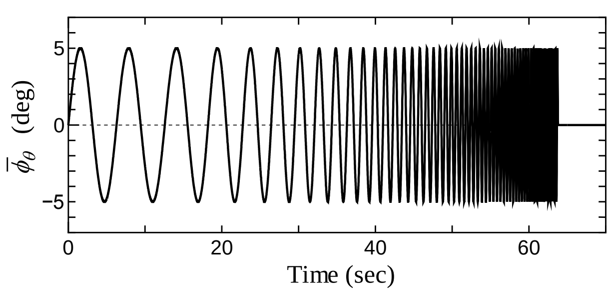


Figure 6 Sweep input for identification of the rotational dynamics

From Equations (5), the transfer functions between the roll/pitch angle and the position of the helicopter are derived as follows:

$$\frac{X}{\Phi_r} = \frac{\gamma_x e^{-\tau_g s}}{s^2 + c_x s}, \quad \frac{Y}{\Phi_p} = \frac{\gamma_y e^{-\tau_g s}}{s^2 + c_y s}. \tag{9}$$

Additional exponential elements in Equation (9) result from the time lag of the GPS measurement. By fitting the result of frequency response tests to the transfer functions in frequency domain, we can obtain the system parameters in Eqs. (5) – (7).

B. Frequency response test for system identification

We implemented frequency response tests for the rotational dynamics and the translational dynamics, respectively. In the experimental helicopter, the frequency ranges were determined as 1.0 - 25 rad/s for the rotational dynamics identification and 0.7 - 5.0 rad/s for the translational dynamics identification.

Frequency-sweep input denoted by Tischler and Remble was given to the reference values in Equations (2) [18]. The frequency-sweep signal based on the exponential function is given by:

$$\begin{aligned} \bar{\phi}_\theta &= A \sin \psi \\ \psi &= \int_0^{T_{rec}} \omega \, dt \\ \omega &= \omega_{min} + K(\omega_{max} - \omega_{min}) \\ K &= 0.0187\{\exp(4t / T_{rec}) - 1\} \end{aligned} \tag{10}$$

where T_{rec} is the duration for the Fourier analysis and A is the amplitude of the reference values. We can set the frequency range by ω_{max} and ω_{min}. Before starting progression in terms of ω in Equation (10), the constant frequency ω_{min} is added for two cycle periods to ensure a good steady state condition. Figure 6 shows the frequency-sweep input for the rotational dynamics whose amplitude was set to 5.0 degrees in the test. In the translational dynamics identification, the references $\bar{\phi}_x$ and $\bar{\phi}_y$ are given by the same function in Equation (10) and the amplitude A was set to 5.0 m.

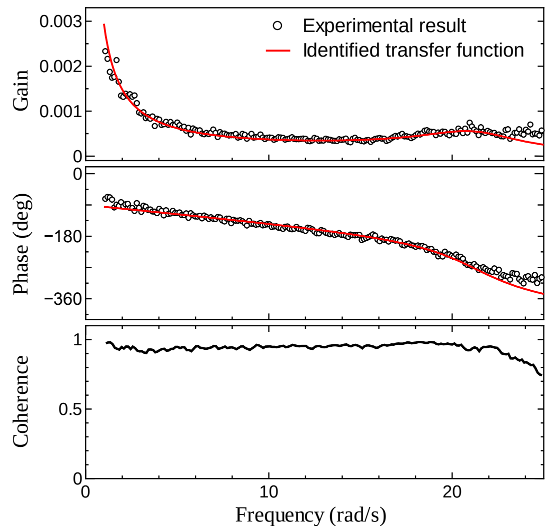
We implemented frequency response tests for the rotational dynamics in terms of roll and pitch, respectively. We set T_{rec} = 51.2 sec and the position control were superposed in the test. Frequency response diagrams derived from the results are plotted in **Figure 7**. The validation of a frequency response test can generally be verified by the coherence function. The value is calculated from the power spectrums and the cross spectrum between input and output as follows:

$$Coh = \frac{|G_{U\Phi}|^2}{G_{UU} \cdot G_{\Phi\Phi}} \tag{11}$$

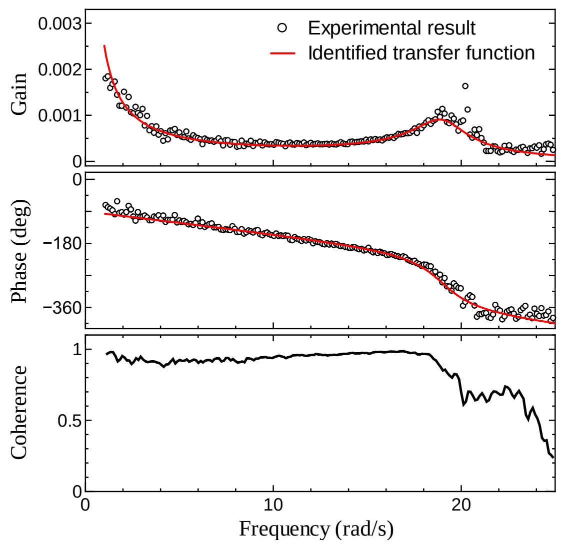
A value of 0.6 for the coherence function is usually used as a limit [17]. Coherence in the results indicates that most of frequencies in the setting frequency range were satisfied with the metric. By using a nonlinear least-squares fitting in frequency domain, a couple of transfer function were calculated as follows:

$$\frac{\Phi_r}{U_r} = \frac{1.36 e^{-0.0808 s}}{s^3 + 5.45 s^2 + 461.5 s}, \quad \frac{\Phi_p}{U_p} = \frac{0.914 e^{-0.104 s}}{s^3 + 2.78 s^2 + 362 s} \tag{12}$$

The red lines in **Figure 7** demonstrate the frequency response of the transfer functions in Equation (12) and they are in good agreement with the experimental results. The system parameters identified from the experimental results are summarized in **Table 1**.

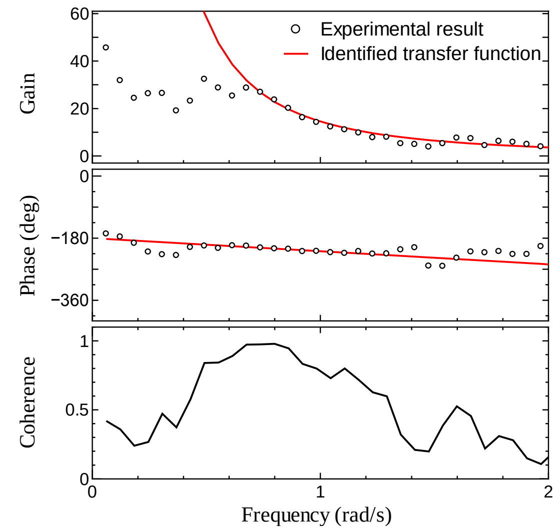


(a) Roll

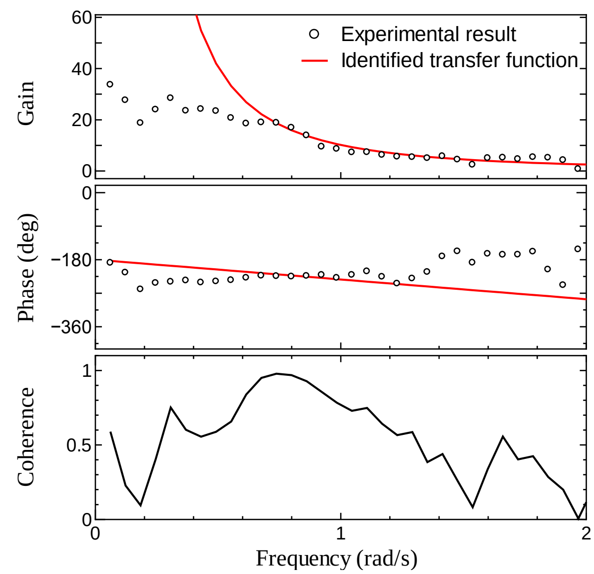


(b) Pitch

Figure 7 Frequency response diagram for the rotational dynamics



(a) Lateral direction



(b) Longitudinal direction

Figure 8 Frequency response diagram for the translational dynamics

Table 1 System parameters for rotational dynamics

	τ_{lr} / τ_{lp}	T_a / T_b	G_a / G_b	α / β
Roll / Lateral	0.0808	0.184	461	5.39×10^{-3}
Pitch / Longitudinal	0.1043	0.357	362	9.00×10^{-3}

Table 2 System parameters for translational dynamics

	τ_g	γ_a / γ_b
Lateral	0.662	14.6
Longitudinal	0.927	10.2

The results of the frequency response test for the translational dynamics are shown in **Figure 8**. We set T_{rec} = 102.4 sec in the test. In the frequency range whose coherence is more than 0.6, fitted transfer functions were obtained as follows:

$$\frac{X_x}{\Phi_r}=\frac{14.6e^{-0.662s}}{s^2},\quad \frac{V_y}{\Phi_p}=\frac{10.2e^{-0.927s}}{s^2} \qquad (13)$$

The results of the frequency response test for the translational dynamics are shown in **Figure 8**. We set T_{rec} = 102.4 sec in the test.

In the frequency range whose coherence is more than 0.6, fitted transfer functions were obtained as follows:

$$\frac{X_x}{\Phi_r}=\frac{14.6e^{-0.662s}}{s^2},\quad \frac{V_y}{\Phi_p}=\frac{10.2e^{-0.927s}}{s^2}, \qquad (13)$$

where V_x and V_y mean Laplace transform of velocity of the translational dynamics. We intentionally neglect the damping of the translational dynamics because we cannot accurately identify c_x and c_y with the instruments. Red lines in **Figure 8** describe the frequency response of the fitted transfer function. The system parameters for the translational dynamics identified from the experimental results are summarized in **Table 2**. Fundamentally, the lag time of position sensing τ_g varies with GPS condition. Therefore we regarded τ_g as 0.80 sec in the controller design.

Since the system parameters of the rotational dynamics and the translational dynamics were obtained, we can design the delayed feedback parameters.

V. DESIGN OF DELAYED FEEDBACK CONTROLLER

A. *Planar Model for Controller Design*

We built a simple planar model shown in **Figure 9** to design the delayed feedback controller. We divided the dynamics of the helicopter with slung load system into roll/lateral dynamics and pitch/longitudinal dynamics and designed the controller. The model is composed of a rigid body (Helicopter) and a point mass (Load) connected by a cable; M and m are the weights of the rigid body and the mass, J is the moment of inertia of the rigid body, l is the cable length, and L is the distance between the center of gravity of the rigid body and the pivot of the cable. We assume that the rigid body can slide on the rail in the horizontal direction and rotate around the center of gravity.

The equation of motion of the roll/lateral dynamics in the model is given by

$$\begin{aligned} &(M+m)\ddot{x} + mL\ddot{\phi}_r - ml\ddot{\theta}_r = M\gamma_x \tan\phi_r \\ &-mL\ddot{x} + (J_r + mL^2)\ddot{\phi}_r + mLl\ddot{\theta} + mgL\phi_r = J_r G_a a \\ &-ml\ddot{x} + mLl\ddot{\phi}_r + ml^2\ddot{\theta}_r + mgl\theta_r = 0. \end{aligned} \tag{14}$$

Table 3 shows the system parameters. From Equations (2), (4), (7) and (14), the corresponding Laplace domain representation leads to the following characteristic equation:

$$\begin{vmatrix} q_{r11} & q_{r12} & q_{r13} \\ q_{r21} & q_{r22} & q_{r23} \\ q_{r31} & q_{r32} & q_{r33} \end{vmatrix} = 0 \tag{15}$$

where

$$\begin{aligned} q_{r11} &= (M+m)s^2 \\ q_{r12} &= -mLs^2 - M\gamma_x \\ q_{r13} &= -mls^2 \\ q_{r21} &= -(T_a s+1)mLs^2 + J_r G_a \alpha k_{pr}(K_{dx}s + K_{px})e^{-(\tau_{lr}+\tau_g)s} \\ q_{r22} &= (T_a s+1)\{(J_r + mL^2)s^2 + mgL\} \\ &\quad + J_r G_a\{T_a s + \alpha(k_{dr}s + k_{pr})e^{-\tau_{lr}s}\} \\ q_{r23} &= (T_a s+1)mLls^2 - J_r G_a \alpha k_{pr} G_{dr} e^{-(\tau_{lr}+\tau_{dr})s} \\ q_{r31} &= -mls^2 \\ q_{r32} &= mLls^2 \\ q_{r33} &= ml^2 s^2 + mgl. \end{aligned}$$

The other characteristic equation of the pitch/longitudinal dynamics can be represented by the same form.

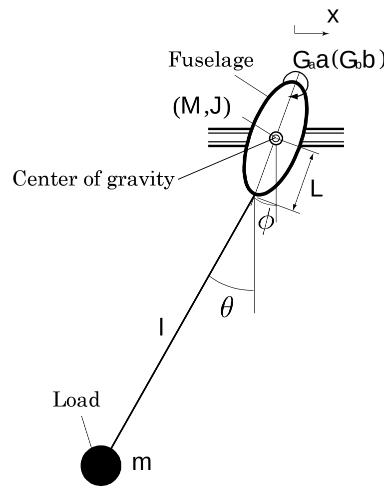


Figure 9 Simple model for the delayed feedback controller design

Table 3 System parameters for controller design

M [kg]	m [kg]	J_r [kgm^2]	J_p [kgm^2]	L [m]	l [m]
6.06	0.73	0.050	0.100	0.20	3.0

Since there are an infinite number of poles in the characteristic equation due to the exponential function, it is hard to calculate all poles. As a measure to this problem, we introduced a 3rd order Padé approximation as follows:

$$e^{-\tau s} = \frac{1 - \frac{1}{2}\tau s + \frac{1}{10}(\tau s)^2 - \frac{1}{120}(\tau s)^3}{1 + \frac{1}{2}\tau s + \frac{1}{10}(\tau s)^2 + \frac{1}{120}(\tau s)^3}. \tag{16}$$

As a result, there are 16 poles in the characteristic equation.

B. *Controller design based on root locus*

We designed the delayed feedback control parameters theoretically based on root locus [19],[20]. As an example of the design strategy, we denote the process for controller design of the roll/lateral dynamics.

Figure 10 shows the root loci of Equation (15) with τ_{dr} = 0.6 sec. Since the poles are conjugate pairs, the root loci are plotted only in the upper half complex plane. The characteristic equation has 16 poles, however, significant poles for the controller design are two couples of complex conjugate; one means vibration of the slung load and the other means translational dynamics of the helicopter. **Figure 10** indicates that increasing G_d improves convergence properties of slung load vibration, but it destabilizes the translational dynamics. The characteristic agrees well with the experimental result. Past studies indicated that a helicopter with slung load system has a trade-off between piloted handling qualities and load damping. We defined the optimum parameter as G_d = 0.027 to make the best trade-off and the poles are plotted by circles in **Figure 10**.

The parameter is in rough agreement with the optimal parameters (G_{dr} = 0.022, τ_{dr} =0.6) determined by trial and error in the experiments.

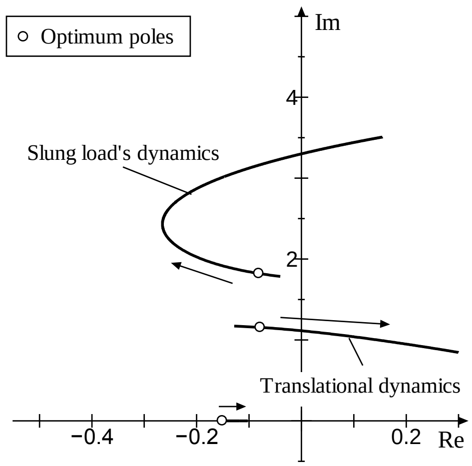


Figure 10 Root locus of the roll/lateral dynamics with τ_{dr} = 0.6 sec

VI. Conclusion

This paper has dealt with the practical vibration control method for a system of a small-size helicopter with a slung load and the controller design strategy. As the first step in this study, we developed the vibration control method for a helicopter with a slung load system and verified the performance of the present control method by flight tests. The present method enables us to suppress the vibration of the slung load just by measuring the angle of the slung load.

We tried to theoretically design the delayed feedback controller as the second step. We identified the rotational and the translational dynamics by frequency response tests and the delayed feedback controller was designed by root locus of the simple specialized model. The optimum delayed feedback parameters derived theoretically are nearly identical to the parameter determined by trial-and-error in the experiment.

This study is an important contribution for transport by using small-size helicopters because the method enables us to suppress vibration of a slung load by mounting just a simple angle measuring device, and we clarified a way to design the delayed feedback controller in the system.

References

[1] L. R. Lucassen and F. J. Sterk, "Dynamic Stability Analysis of a Hovering Helicopter with a Sling Load," *Journal of the American Helicopter Society*, Vol. 10, No. 2, 1965, pp. 6-12. CrossRef

[2] T. A. Dukes, "Maneuvering Heavy Sling Loads Near Hover Part I: Damping the Pendulous Motion," *Journal of the American Helicopter Society*, Vol. 18, No. 2, 1973, pp. 2-11. CrossRef

[3] T. A. Dukes, "Maneuvering Heavy Sling Loads Near Hover Part II: Some Elementary Maneuvers," *Journal of the American Helicopter Society*, Vol. 18, No. 3, 1973, pp. 17-22. CrossRef

[4] C. Poll and D. Cromack, "Dynamics of Slung Bodies Using a Single-Point Suspension System," *Journal of Aircraft*, Vol. 10, No. 2 1973, pp. 80-86.

[5] L. S. Cicolani, A. Cone, J. N. Theron, D. Robinson, J. Lusardi, M. B. Tischler, A. Rosen, and R. Raz, "Flight Test and Simulation of a Cargo Container Slung Load in Forward Flight," *Journal of the American Helicopter Society*, Vol. 54, No. 3, 2009. CrossRef

[6] R. Sahai, L. S. Cicolani, M. B. Tischler, C. L. Blanken, C. C. Sullivan, M. Y. Wei. Y. Ng, and L. E. Pierce, "Flight-time identification of helicopter-slung load frequency response characteristics using CIFER", presented at AIAA Guidance, Navigation, and Control Conference, Portland, Oregon, 1999. AVAILABLE

[7] C. M. Ivler, M. B. Tischler, J. D. Powell, "Cable angle feedback control systems to improve handling qualities for helicopters with slung loads," presented at AIAA Guidance, Navigation, and Control Conference, Portland, Oregon, 2011.

[8] C. M. Ivler, J. D. Powell, M. B. Tischler, J. W. Fletcher, J.W., and C. Ott, "Design and flight test of a cable angle/rate feedback flight control system for the RASCAL JUH-60 helicopter," presented at the American Helicopter Society 68th Annual Forum, Fort-Worth, Texas, 2012.

[9] T. Oktay and C. Sultan, "Modeling and control of a helicopter slung-load system," *Aerospace Science and Technology*, Vol. 29, Issue 1, 2013, pp. 206-222. CrossRef

[10] M. Bernard, K. Kondak, and G. Honmel, "Load transportation system based on autonomous small size helicopters," *The aeronautical journal*, Vol. 114, No. 1153, 2010, pp.191-198.

[11] M. Bisgaard, A. Cour-Harbo, and J. D. Bendtsen, "Full State Estimation for Helicopter Slung Load System," presented at AIAA Guidance, Navigation, and Control Conference, Hilton Head, South Carolina, 2007. CrossRef

[12] M. Bisgaard, A. Cour-Harbo, and J. D. Bendtsen, "Swing damping for helicopter slung load systems using delayed feedback," presented at AIAA Guidance, Navigation, and Control Conference, Chicago, 2009. CrossRef

[13] M. Bisgaard, A. Cour-Harbo, and J. D. Bendtsen, "Adaptive control system for autonomous helicopter slung load operations," *Control Engineering Practice*, Vol. 18, No. 7, 2010, pp.800-811. CrossRef

[14] M. Bisgaard, J. D. Bendtsen, and A. Cour-Harbo, "Modeling of generic slung load system," *Journal of Guidance, Control, and Dynamics*, Vol. 32, No. 2, 2009, pp. 573-585. CrossRef

[15] C. J. Adams, "Modeling and control of helicopters carrying suspended loads," Master thesis, Georgia Institute of Technology, 2012.

[16] M. Miwa, K. Kinoshita, and K. Tokuda, "Evaluation of Remote Control Support System for R/C Helicopter," presented at SICE Annual Conference, Taipei, 2010, pp.3112-3115.

[17] B. Mettler, M. B. Tischler, and T. Kanade, "System identification modeling of a small-scale unmanned rotorcraft for flight control design," *Journal of the American helicopter society*, Vol. 47, No. 1, 2002, pp.50-63.

[18] M. B. Tischler and R. M. Remble, *Aircraft and rotorcraft system identification: Engineering methods with flight test examples*, AIAA education series.

[19] Z. N. Masoud, and A. H. Nayfeh, "Sway reduction on container cranes using delayed feedback controller," *Nonlinear dynamics*, Vol.34, No.3-4, 2003, pp. 347-358. CrossRef

[20] F. E. Udwadia and P. Phohomsiri, "Active control of structures using time delayed positive feedback proportional control designs," *Structural Control and Health Monitoring*, Vol. 13, No.1, 2006, pp.536-552. CrossRef