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Joint Decision on Supplier Selection and Trajectory Tracking Control of Multi-Product Inventory System

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Abstract— In this paper, we develop a mathematical model to determine the optimal joint decision for an integrated supplier selection and trajectory tracking control of multi-product inventory system. We develop two models which are a model in an integer quadratic programming for a case when all parameters including the demand of all products are certain and a model in a stochastic integer quadratic programming for a case when the at least one parameter is uncertain (in this research, at least a demand parameter is uncertain with some probability distribution). Some numerical experiments are given to evaluate the models with randomly generated supply chain data. We used multi-stage programming to solve the corresponding optimization problem. From the results, for all cases, the joint decision is obtained i.e. the optimal product volume purchased from each supplier and the inventory level of each product followed the reference level given by the decision maker.

Keywords— multi-stage programming, supplier selection, inventory control

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I. Introduction

MANAGING inventory of product and selecting the optimal supplier affect the cost spend by the manufacturer, as a consequence, inventory control and supplier selection are two of many important components in logistics and supply chain management [1]. Inventory system needs to be controlled in order to satisfy the demand of the product and minimize the holding cost or it can be controlled for product savings. A mathematical model is the most useful approach to solve the inventory control problem and supplier selection. This mathematical model can be classified into two classes which are deterministic model and stochastic model. Deterministic model is occurred when all parameters are known with certainty whereas stochastic model is occurred when at least one parameter becomes uncertain.

Some researchers were developed a model to solve supplier selection problem by using some optimization method without controlling the inventory level such as mixed-integer model [2] and fuzzy model [3]. Some researchers were integrated the inventory system and supplier selection and solve them at once such as developing a multi echelon MINLP model [4][5], MILP model and heuristic algorithm [6][7], genetic algorithm approach [8], model with lateral transshipments [9] and using a control method [10]. Some researchers were developed the model in deterministic class whereas other researchers were developed in stochastic class but these approaches were not developed the model for trajectory tracking control of the stock of the product. In some case, the decision maker decides to store an amount of a product in his storage for demand satisfying and product saving. How the optimal strategy so that the inventory level will close to a reference level decided by the decision maker is a new problem.

In optimization theory, multi-stage programming refers to a method that can be used to solve a mathematical optimization by dividing the problem into several stages and several simpler sub-problems and solving them recursively [12]. For stochastic multi-stage programming, the problem can be solved by generating the scenarios based on the probability distribution of the random variable(s). The generating scenario can be illustrated by Figure 1 [13]. Suppose that t denotes the stage of the problem, x_t denotes the decision variable at stage t and Ω denotes the event space at stage t. The scenarios are generated based on the probability distribution of event. Firstly, the initial decision is taken in stage 0. The recourse decisions for each stage 1; 2; ⋯; T is taken in succeeding stages as corrective decisions based on the past decisions and the realization of the actual values of the uncertain parameters. Then, the possible outcomes can be expressed as a scenario presenting one possible realization of the future and the enumeration of all possible combinations of outcomes.

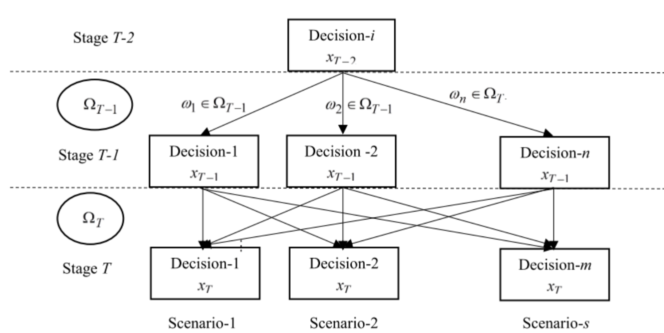

Figure 1 Scenario tree of a dynamic stochastic programming

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In the past works, we were solved the integrated inventory control and supplier selection problem for single product case [11][14]. In this work, we develop a mathematical model to solve an integrated multi-product inventory control problem and its supplier selection problem for multi-product inventory system. We develop a model in deterministic environment where all parameters are certain and a model stochastic environment where at least one parameter, demand in this work, is uncertain with some probability distribution. We solve the mathematical problem by using multi-stage programming and give some numerical experiments to evaluate the model.

II. Problem Definition

Two problems in LSCM which are supplier selection problem and inventory control problem will be solved simultaneously. Supplier selection problem is occurred when a manufacturer will purchase a material or product from several suppliers and the method should determine the optimal product volume that will be purchased from each supplier so that the total purchasing cost will be minimized. Inventory control problem is occurred when the decision maker decides to control the product volume in the storage so that the inventory level will be located at some point as close as possible to a reference level decided by the decision maker with minimal cost. Integrating these problems and solve it simultaneously is a good idea in order to find the best strategy for whole problems. The developed model will work for multi-supplier, multi-product and multi-period. The model will also work in unknown environment where some parameters are uncertain with some probability distribution. To develop the mathematical model, we have several assumptions which are the model is work for non-perishable product or perishable product but it will be not expired along the optimization time period.

Table 1 Parameters and variables of the model

Symbol	Description
s	Index of supplier
p	Index of product
t	Index of time period
$X_{t,s,p}$	Amount of product p purchased from supplier s at time period t (unit)
$I_{t,p}$	Inventory level of product p at time period t (unit)
$U_{t,s,p}$	Purchasing cost per unit product p from supplier s at time period t
$H_{t,p}$	Holding cost per unit product p at time period t
$D_{t,p}$	Demand of product p at time period t (unit)
$r_{t,p}$	Reference inventory level for control purposes of product p at time period t (unit)
$C_{t,p}$	Supplier capacity of supplier s for product p (unit)
$M_{t,p}$	Storage capacity of product p at time period t (unit)
Ω	Event space of the problem in probabilistic environment
P_{r1}	Probability of scenario i

III. Mathematical Model

Suppose that a manufacturer will purchase a product or material from S supplier alternatives. We propose a mathematical model that minimize the total procurement cost containing purchasing cost and holding cost. The parameters and variables that will be used are summarized in Table 1. We define a track reference objectives as $(I_t - r_t)^2$, *i.e.*, the gain of the inventory level to the reference level.

A. Deterministic environment

For deterministic environment, the objective function J is defined as follows

$$J = \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{p=1}^{P} U_{t,s,p}X_{t,s,p} + \sum_{t=1}^{T}\sum_{p=1}^{P} H_{t,p}I_{t,p} + \sum_{t=1}^{T}\sum_{p=1}^{P}\left(I_{t,p} - r_{t,p}\right)^2 \quad \textbf{(1)}$$

where the first term defines the total purchasing cost of all products from all suppliers for all time periods, the second term defines the total holding cost of all products for all time periods and the third term defines the track reference objectives.

The constraints are defined as follows. The first constraint is used to ensure that the demand of each product is satisfied for all time periods that defined as follows

$$\left.\begin{array}{c} I_{t-1,p} + \sum_{s=1}^{S} X_{t,s,p} - I_{t,p} \ge D_{t,p}, \\ \forall t \in \{1,2,\cdots,T\},\ \forall p \in \{1,2,\cdots,P\} \end{array}\right\} \quad \textbf{(2)}$$

The second constraint ensures that the purchased product volume from supplier s will not exceed the maximum capacity, *i.e.*

$$\left.\begin{array}{c} X_{t,s,p} \le C_{t,s,p}, \\ \forall t \in \{1,2,\cdots,T\},\ \forall s \in \{1,2,\cdots,S\},\ \forall p \in \{1,2,\cdots,P\} \end{array}\right\} \quad \textbf{(3)}$$

The third constraint ensures that the inventory of the product will not exceed the maximum capacity of the storage, *i.e.*

$$\left.\begin{array}{c} I_{t,p} \le M_{t,p}, \\ \forall t \in \{1,2,\cdots,T\},\ \forall p \in \{1,2,\cdots,P\} \end{array}\right\} \quad \textbf{(4)}$$

The last constraint is integer constraint for product volume, *i.e.*

$$X_{t,s,p} \in \{0,1,2,\cdots\} \quad \textbf{(5)}$$

By rewriting (1)-(5) we have the first mathematical model, i.e., model for deterministic environment by minimizing the total as follows

$$\min\left(\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{p=1}^{P} U_{t,s,p}X_{t,s,p} + \sum_{t=1}^{T}\sum_{p=1}^{P} H_{t,p}I_{t,p} + \sum_{t=1}^{T}\sum_{p=1}^{P}\left(I_{t,p} - r_{t,p}\right)^2\right) \quad \textbf{(6)}$$

subject to

$$I_{t-1,p} + \sum_{s=1}^{S} X_{t,s,p} - I_{t,p} \ge D_{t,p}, \quad \forall t \in \{1,2,\cdots,T\}, \quad \forall p \in \{1,2,\cdots,P\} \quad \textbf{(2)}$$

$$X_{t,s,p} \le C_{t,s,p}, \quad \forall t \in \{1,2,\cdots,T\},\ \forall s \in \{1,2,\cdots,S\}, \quad \forall p \in \{1,2,\cdots,P\} \quad \textbf{(3)}$$

$$I_{t,p} \le M_{t,p}, \quad \forall t \in \{1,2,\cdots,T\},\ \forall p \in \{1,2,\cdots,P\} \tag{4}$$

$$X_{t,s,p} \in \{0,1,2,\cdots\} \tag{5}$$

B. *Probabilistic environment*

The second model is formulated in probabilistic environment where at least one parameter becomes uncertain where this uncertainty is approached by using some probability distribution. Assume that the event space of the random variable is Ω. We will solve the problem by using multi-stage stochastic programming by generating the scenario of the problem based on the event space. We formulate the model as minimizing the expected total cost, *i.e.*

$$\min \sum_{i=1}^{\Omega}\left(\Pr_i \cdot \left(\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{p=1}^{P} U_{t,s,p}X_{t,s,p} + \sum_{t=1}^{T}\sum_{p=1}^{P} H_{t,p}I_{t,p} + \sum_{t=1}^{T}\sum_{p=1}^{P}\left(I_{t,p} - r_{t,p}\right)^2\right)\right) \tag{7}$$

where the constraints are following the first model i.e. model in deterministic environment.

IV. Numerical Experiment

In this section, we give three example to evaluate the model. For the first example, we evaluate the problem in deterministic environment where all parameters are known with certainty. For the second example, we evaluate the model in unknown environment where the demand is random. The last example will give the sensitivity analysis to analyze the impact of the demands uncertainty. For each example, the decision maker desires that the inventory level must be as close as possible to a reference point defined by the decision maker that will appear in the corresponding example. We solved the optimization problem in Windows 8 of OS, AMD A6 2.7GHz of processor and 2 GB of memory by using software LINGO 16.0 where the optimization model class multi-stage quadratic programming.

<u>Example 1:</u> Deterministic environment

Suppose that a manufacturer has three suppliers namely s_1, s_2 and s_3 that will supply three products namely p_1, p_2 and p_3 where the purchasing cost for all suppliers are given in Table 2 and the supplier maximum capacity for each product is given in Table 3. Other parameters are given in Table 4. We solved the problem for 10 time periods. The demand for each product is given in Figure 2. By solving (6), the solution of the problem, *i.e.*, the optimal strategy for each time period is given in Figure3. The inventory level of each product will be controlled so that it will be as close as possible to a reference level given in Figure4.

Figure 3 shows the optimal strategy of the problem, *i.e.*, the optimal product volume purchased from each supplier for each time period. It can be seen that for time period 1, the manufacturer must purchase 30 units of p_1 from supplier s_1, 27 units of p_2 from s_1, 11 units of p_3 from s_1, 47 units of p_3 from s_2, 78 units of p_1 from s_3, 10 units of p_2 from s_3 and 20 units of p_3 from s_3. Since the demand of p_1, p_2 and p_3 at time period 1 are 280 units, 271 units and 311 units respectively then the manufacturer has 98 units of p_1, 109 units of p_2 and 47 units of p_3 in the storage where the decision maker decided the reference stock level for p_1, p_2 and p_3 are 100 units, 110 units and 50 units respectively. The actual inventory level of all products and their reference level are given in Figure 4 and it can be seen that the actual stock of all products are sufficiently closed to their reference levels.

Table 2 Purchasing cost

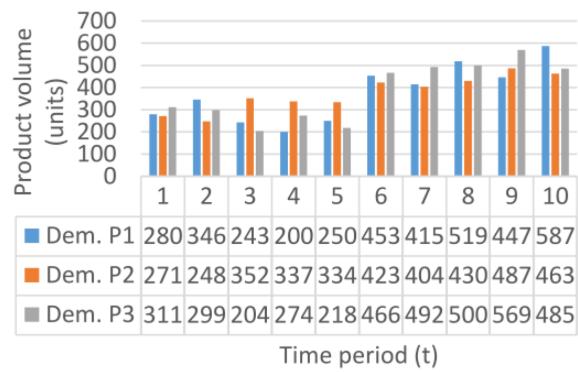
Supplier	p_1	p_2	p_3
s_1	50	20	100
s_2	52	21	101
s_3	50	21	99

Table 3 Supplier capacity (units)

Supplier	p_1	p_2	p_3
s_1	300	400	500
s_2	500	200	500
s_3	250	250	200

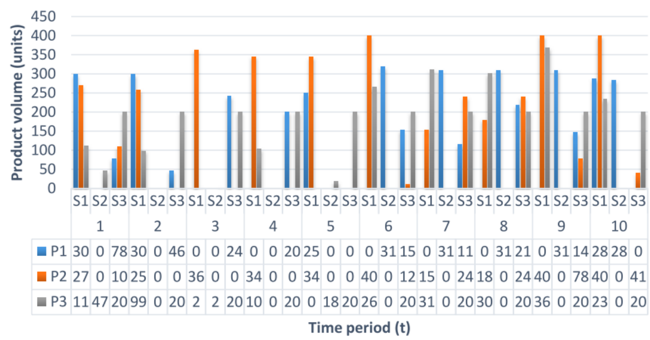
Table 4 Inventory capacity and holding cost

Supplier	p_1	p_2	p_3
Inventory capacity (units)	300	250	250
Holding cost ($)	4	2	3



	1	2	3	4	5	6	7	8	9	10
Dem. P1	280	346	243	200	250	453	415	519	447	587
Dem. P2	271	248	352	337	334	423	404	430	487	463
Dem. P3	311	299	204	274	218	466	492	500	569	485

Figure 2 Demand value for Example 1
(P1, P2 and P3 refers to p_1, p_2 and p_3)



	1 S1	1 S2	1 S3	2 S1	2 S2	2 S3	3 S1	3 S2	3 S3	4 S1	4 S2	4 S3	5 S1	5 S2	5 S3
P1	30	0	78	30	0	46	0	0	24	0	0	20	25	0	0
P2	27	0	10	25	0	0	36	0	0	34	0	0	34	0	0
P3	11	47	20	99	0	20	2	2	20	10	0	20	0	18	20

	6 S1	6 S2	6 S3	7 S1	7 S2	7 S3	8 S1	8 S2	8 S3	9 S1	9 S2	9 S3	10 S1	10 S2	10 S3
P1	0	31	15	0	31	11	0	31	21	0	31	14	28	28	0
P2	40	0	12	15	0	24	18	0	24	40	0	78	40	0	41
P3	26	0	20	31	0	20	30	0	20	36	0	20	23	0	20

Figure 3 Solution for Example 1
(S1, S2 and S3 refers to s_1, s_2 and s_3)

	1	2	3	4	5	6	7	8	9	10
Inv. P1	98	98	98	98	99	118	128	138	148	132
Inv. P2	109	119	130	139	150	139	129	119	110	88
Inv. P3	47	47	47	77	77	78	97	98	98	48
Ref. P1	100	100	100	100	100	120	130	140	150	160
Ref. P2	110	120	130	140	150	140	130	120	110	100
Ref. P3	50	50	50	80	80	80	100	100	100	100

Figure 4 Inventory level and its reference level for p_1, p_2 and p_3

Example 2: Stochastic environment

In this example, we solve the problem in stochastic environment. Due to the computers capacity limit, we solve the problem with 2-by-2 time periods, only demand for p_1 that is random and its probability distribution has only two possible event. Let the demands probability distribution for p_1 is given in Table 5 and demand for p_2 and p_3 are assumed to be certain that are $D_{t,2} = 300$ and $D_{t,3} = 250$ for all t.

The remaining variables are following the Example 1. Firstly, we solve (7) for only 2 time periods which has 9 scenarios illustrated by Table 6. We solve this problem by using stochastic multi-stage programming in LINGO 16.0 where the model class is quadratic integer programming.

The expected total cost, *i.e.*, the objectives value of (7) is $0.09 \cdot \$97960 + 0.09 \cdot \$102959 + 0.12 \cdot \$107958 + \cdots + 0.16 \cdot \$118055$ $= \$109070$. From scenario tree given in Table6 and Figure 5, the optimal decision for each time period can be obtained after the random variables at the corresponding time period are revealed. Let the demand for (p_1, p_2, p_3) is (200, 300, 250) units. From Figure 5, it can be seen that the solution for time period 1 is purchase 198 units of p_1 from supplier s_1, 400 units of p_2 from s_1, 98 units of p_3 from s_1, 200 units of p_1 from s_3, 8 units of p_2 from s_3, and 200 units of p_3 from s_3. It will give (198, 108, 48) units of (p_1, p_2, p_3) in the storage. The optimal decision for time period 2 can be obtained after the the demand for p_1 is revealed. Let the demand for p_1 at time period 2 is 200 units, then the optimal decision is (0, 301, 0) units, (0, 0, 2) units and (175, 0, 200) units of (p_1, p_2, p_3) from $(s_1, s_2$, and $s_3)$ respectively.

At each time period, after the demand value of p_1 is revealed, we can determine the optimal strategy for the corresponding time period. We evaluate 3.7 for 12 time periods to observe the actual stock level and the reference level that illustrated by Figure 6. It can be seen that for time periods 1, 3, 5, ..., 11, the actual stock level is sufficiently closed to the reference level but for time periods 2, 4, 6, ..., 12, the difference between the actual stock level and the reference level are sufficiently large especially for p_3. We observe that it was caused by 2-by-2 time periods evaluating. For each 2 time periods evaluating, it assuming that the problem is only optimized for 2 time periods and the model decides not adding the inventory level. From this result, we observe that if the model is evaluated for 12 time periods in one time then the actual stock level and the reference are sufficiently closed as well as the result in Example 1.

Example 3: Sensitivity analysis

In this example, we solve the problem in stochastic environment under assumption that the demand of each product is normally distributed. By using various standard deviations, we solve the problem to analyze the impact of the demands uncertainty to the expected total cost. Let the mean of the demand of each product is 200 and the standard deviation is $\sigma = 20, 30, 50, 70, 100,$ and 150. We use the Monte Carlo sampling to approach these continuous probability distributions with finite discrete sample where the sample size is 3 for each random variable and each time period. The result is given in Figure 7. From the result, it can be seen that the larger the standard deviation of the random variable, the larger the expected total cost. It was caused by the wide range of the uncertainty of the random variable.

Table 5 Demands probability distribution for $D_{t,1}$

Probability	Demand (unit)
0.7	200
0.3	300

Table 6 Scenario tree for Example 2

Scenario	Time period (t)	Demand p_i (units)	Solution	Inventory (units)	Probability	Total cost ($)
1	1	(200,300,250)	Fig.5.a	(19,810,848)	0.09	97,960
	2	(200,300,250)		(1,731,090)		
2	1	(200,300,250)	Fig.5.b	(19,810,848)	0.09	102,959
	2	(300,300,250)		(1,731,100)		
3	1	(200,300,250)	Fig.5.c	(19,810,848)	0.12	107,958
	2	(400,300,250)		(1,731,090)		
9	1	(400,300,250)	Fig.5.d	(19,810,847)	0.12	118,055
	2	(400,300,250)		(1,731,080)		

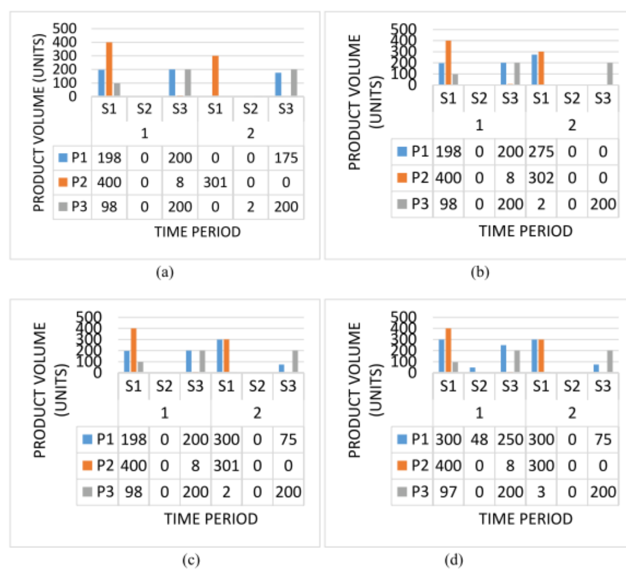


Figure 5 Solution of Example 2

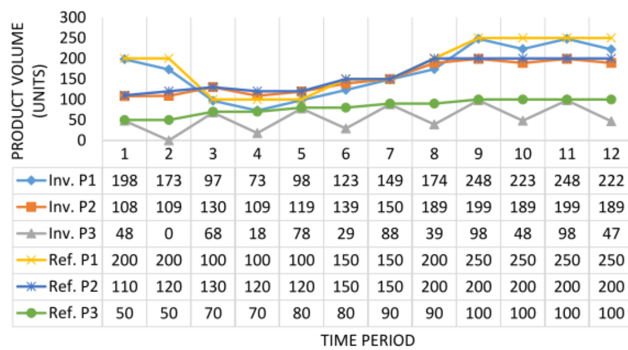


Figure 6 Inventory level for each product and its reference level

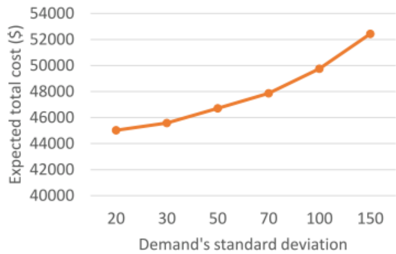


Figure 7 Impact of demands standard deviation to the expected total cost

V. Concluding Remarks

In this paper, an integrated supplier selection problem and trajectory tracking control problem of a multi-product multi-supplier and multi-period inventory system was considered. A mathematical model was formulated each for a problem in deterministic environment and stochastic environment. Numerical experiments were considered with randomly generated data for model evaluation. From the results, it can be concluded that the supplier selection problem and trajectory tracking control problem of this inventory system were solved simultaneously which called integrated solution or joint decision. For each time period, the optimal product volume from each supplier was determined and the inventory level of each product followed the reference level.

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