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Modeling of Multi-Agent System: Equations Appearing in Collision Avoidance Control Methods

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Abstract— Three mainstream approaches of collision avoidance control is discussed: the potential-driven approach, the gyroscopic action approach, and velocity obstacle approach. It is found that the potential-driven approach may also deal with angular motion through the utilization of angular potential field. It is also found that the gyroscopic action approach is actually the angular aspect of the potential-driven approach. The formulation of control solution using velocity obstacle is presented, along with its geometric representation. So far in the discussion, control solution using velocity obstacle approach is most resistant to deadlock situation.

Keywords— collision avoidance, potential-driven, gyroscopic action, velocity obstacle

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I. Introduction

Collision is a condition where two bodies occupy the same region in physical space at the same time. In reality, such condition never actually occur since it always preceded by an impact that generates a pair of repulsive forces in the direction that generally separates the two bodies. Such repulsive forces, in most cases, have damaging effect on the two bodies involved in that collision. Therefore, the study about collision is important since preventing collision means preventing damage, which in turn, saving cost and even lives.

In studies about vehicle control system, collision condition is one of important cases that must be considered, especially in designing control system for vehicles that will navigate in environment with many uncertainties. Among other things, such uncertainties may be the number of obstacles, the geometric size of obstacle, the state of motion of the obstacle, whether it is static or moving, how it behaves when it moves, etcetera.

This paper will discuss three mainstream strategies of collision avoiding (CA) control system for autonomous vehicles: potential-driven CA [1]-[3], CA by gyroscopic action [4]-[6], and CA by velocity obstacle vector set [7]. In the discussion, we try to capture the core idea behind each method that may not be apparent in the original works.

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II. Potential-driven Collision Avoidance Control

The general idea in designing a CA control system is to bring the controlled body away from any identified collision threat, and potential-driven CA is known to be the most straight-forward method in doing this. This method is also known to be cost-effective in term of computational cost since all the process can generally be carried out using simple mathematical operation.

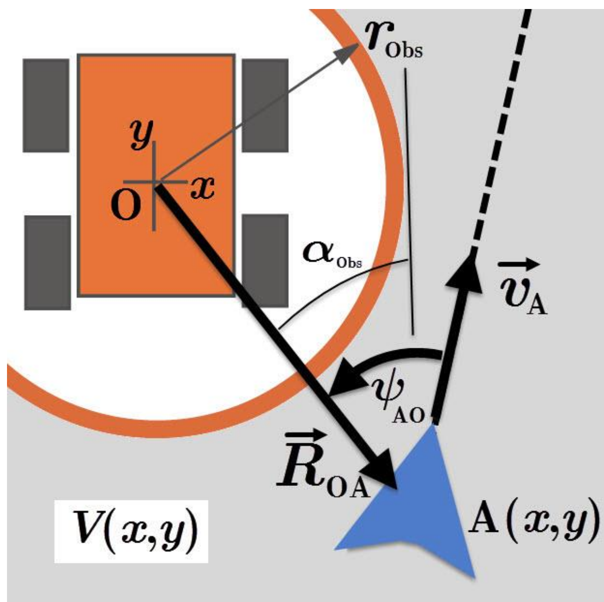

Figure 1 Potential field *V* around an obstacle body with geometric center at O, and a controlled vehicle A in that field

In potential-driven CA, a virtual potential field (1) is defined around a body that is identified as obstacle, typically with the body's geometric center as the source of the field. Then, to a vehicle venturing into this field, the control action for this vehicle to avoid colliding with the obstacle is formulated as a function of the strength of that potential field. Figure 1 shows a typical geometric model of vehicle-obstacle system that is used to formulate CA control action.

$$V := \frac{1}{\left\|\vec{R}_{OA}\right\|} \tag{1}$$

Equation (1) is open to be modified to consider obstacle's size, for example, by introducing the effective radius of the obstacle r_{Obs}.

$$V := \frac{1}{\left\|\vec{R}_{\text{OA}}\right\| - r_{\text{Obs}}}, \quad \vec{R}_{\text{OA}} = \vec{R}_{\text{Obs}} - \vec{R}_{\text{A}} \tag{2}$$

Then, the CA control action must bring the controlled vehicle to places where the value of the potential is lower (3).

$$\frac{dV}{ds} < 0, \quad s := \left\|\vec{R}_{\text{OA}}\right\| - \left\|\vec{R}_{\text{obs}}\right\| \tag{3}$$

And the CA control action can be formulated as (4) to satisfy (3).

$$\vec{u}_{\text{CA}} = k \cdot V \cdot \frac{1}{\left\|\vec{R}_{\text{OA}}\right\|} \cdot \vec{R}_{\text{OA}} \tag{4}$$

Equation (4) can be expanded to (5) to enhance control performance and facilitate further analysis:

$$\left.\begin{aligned} \vec{u}_{\text{CA}} &= \left(k_1 \cdot V + k_2 \cdot \frac{dV}{dt} + k_3 \cdot \frac{d^2V}{dt^2}\right) \cdot \vec{1}_{\text{OA}} \\ \vec{1}_{\text{OA}} &:= \frac{\vec{R}_{\text{OA}}}{\left\|\vec{R}_{\text{OA}}\right\|} \end{aligned}\right\} \tag{5}$$

The stiffness, damping, and inertial coefficients (k_1, k_2, and k_3) can be optimized to meet with the pre-defined control requirements.

Since (4) only deals with translational distance, this controller is only applicable when the controller's degree of freedom (DoF) is not less than the controlled system's DoF, *i.e.*, the overall system is holonomic. For non-holonomic systems, CA action that must work in the direction of constrained DoF is worked around by applying control action in another DoF that is not constrained, which may include angular motion DoF. For such cases, similar formulation can be done in term of relative angular distance or relative orientation that is defined as the angular distance of the line of sight of the obstacle with respect to vehicle's velocity direction.

$$V_{\text{trans}} := \frac{1}{s}, \qquad V_{\text{ang}} := \frac{1}{\varphi} \tag{6}$$

with

$$\left.\begin{aligned} s &:= \left\|\vec{R}_{\text{OA}}\right\|, & \varphi &:= \psi_{\text{AO}} - \alpha_{\text{Obs}} \\ \psi_{\text{AO}} &= \arccos\left(-\vec{1}_{\vec{R}_{\text{OA}}} \cdot \vec{1}_{\vec{v}_{\text{OA}}}\right), & \alpha_{\text{Obs}} &= \arcsin\left(\frac{r_{\text{Obs}}}{\left\|\vec{R}_{\text{OA}}\right\|}\right) \\ \vec{1}_{\vec{R}_{\text{OA}}} &:= \frac{1}{\left\|\vec{R}_{\text{OA}}\right\|} \cdot \left\|\vec{R}_{\text{OA}}\right\|, & \vec{1}_{\vec{v}_{\text{OA}}} &:= \frac{1}{\left\|\vec{v}_{\text{OA}}\right\|} \cdot \left\|\vec{v}_{\text{OA}}\right\| \end{aligned}\right\} \tag{7}$$

then

$$\vec{u}_{\text{CA}} = \vec{u}_{\text{CA,trans}} + \vec{u}_{\text{CA,ang}} \tag{8}$$

with

$$\left.\begin{aligned} \vec{u}_{\text{CA,trans}} &= \left(k_1 \cdot V_{\text{trans}} + k_2 \cdot \frac{dV_{\text{trans}}}{dt} + k_3 \cdot \frac{d^2V_{\text{trans}}}{dt^2}\right) \cdot \vec{1}_{\vec{R}_{\text{OA}}} \\ \vec{u}_{\text{CA,ang}} &= \left(k_4 \cdot V_{\text{ang}} + k_5 \cdot \frac{dV_{\text{ang}}}{dt} + k_6 \cdot \frac{d^2V_{\text{ang}}}{dt^2}\right) \cdot \vec{1}_{\psi} \\ \vec{1}_{\psi} &:= \frac{\vec{R}_{\text{OA}} \times \vec{v}_{\text{A}}}{\left\|\vec{R}_{\text{OA}} \times \vec{v}_{\text{A}}\right\|} \end{aligned}\right\} \tag{9}$$

The angular part of (6)-(9) resembles another CA approach called CA by gyroscopic action, which will be discussed in the next section.

In cases where multiple obstacles exist, (8) can be applied for each identified obstacle, and summing all the solution to obtain overall control action.

$$\left.\begin{aligned} \vec{u}_{\text{CA}} &= \sum_{j=1}^{N_{\text{Obs}}} \vec{u}_{\text{CA},j} \\ \vec{u}_{\text{CA},j} &= \vec{u}_{\text{CA,trans},j} + \vec{u}_{\text{CA,ang},j} \end{aligned}\right\} \tag{10}$$

(a)

(b)

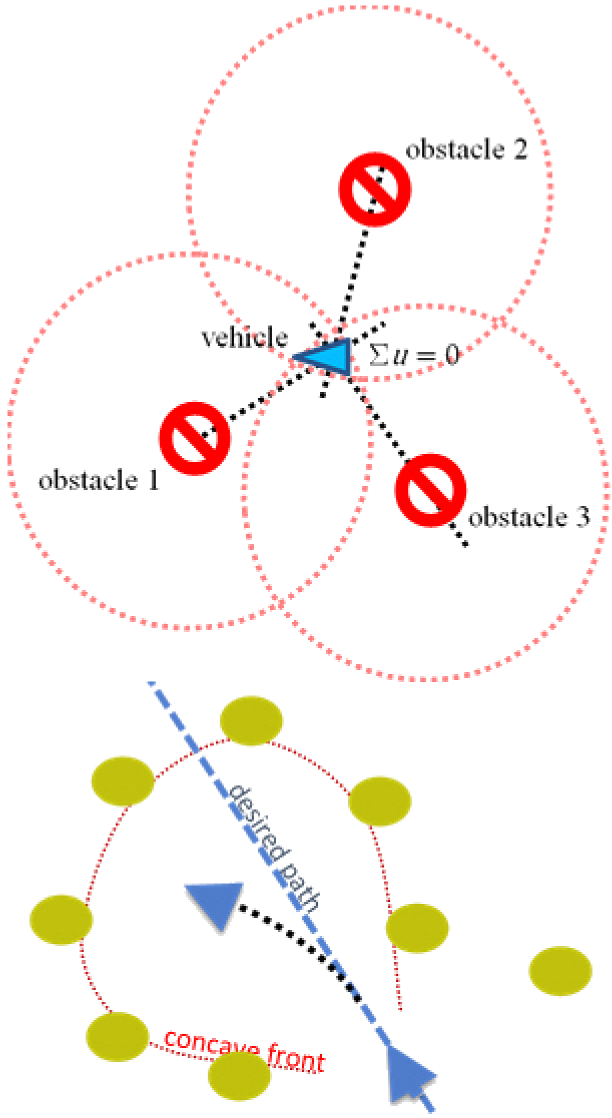

Figure 2 Typical situations where local minima occurs

Equation (10) may result in a value that is equal to but in the opposite direction of another control solution that is being active at the same time, e.g. control for tracking purpose, resulting a zero-magnitude vector. This situation may occur when the vehicle is still in the obstacle's repulsing zone, that is, a zone where CA controller is set to be active. In such case, the vehicle is said to be in deadlock situation, or a local minima condition (11). Such situations usually occur when multiple potential sources form a concave front that it would seem the vehicle is trapped in a hole (Figure 2).

$$\sum_{j=1}^{N_{\mathrm{Obs}}} \frac{dV}{ds_j} = 0 \quad \textbf{(11)}$$

This is a major disadvantage of potential-driven CA approach. CA controller (8) usually gets appended with additional rule that regulates the activation of CA controller to 'tip ooff' vehicle's state out of the local minima.

As can be seen in (6)-(9), the control strategy will work effectively as long as obstacle's position R_{Obs}, obstacle's size r_{Obs}, which may include point obstacle, vehicle's position R_{A}, and vehicle's velocity direction $\frac{\vec{v}_{\mathrm{A}}}{\|\vec{v}_{\mathrm{A}}\|}$ are measurable, or can be estimated by certain means.

III. Collision Avoidance Control by Gyroscopic Action

This control approach is intended to be used specifically in control systems that do not have translational DoF. Therefore, the control action that is derived by this approach is in the form of angular motion. The term gyroscopic is originated from the analogy between how the control action is formulated with the apparent reaction of a spinning gyro when being subjected to a torsional load in the direction that is not aligned with its spin axis (Figure 3). This behavior is governed by (13) which can be derived from Newton's 2nd law for angular motion (12).

$$\frac{d}{dt}\left(\boldsymbol{J}\cdot\vec{\omega}\right) = \vec{M} \quad \textbf{(12)}$$

$$\left(\boldsymbol{J}\cdot\vec{\Omega}\right)\times\vec{\omega} = \vec{M} \quad \textbf{(13)}$$

with $\boldsymbol{J}$ is gyro's tensor of moment inertia. More detailed information regarding mechanics of gyro motion can be found in [8].

Based on the analogy described in Figure 4, a vehicle-obstacle system can be constructed geometrically (Figure 5) to formulate CA controller $\vec{u}_{\mathrm{CA}}$, which is analogous to $\vec{w}$ in (13).

$$\vec{u}_{\mathrm{CA}} = \vec{v}_{\mathrm{A}} \times \frac{1}{\|\vec{R}_{\mathrm{OA}}\|}\cdot\vec{1}_{\vec{R}_{\mathrm{OA}}} = \|\vec{v}_{\mathrm{A}}\|\cdot\frac{1}{\|\vec{R}_{\mathrm{OA}}\|}\cdot\cos\psi_{\mathrm{OA}}\cdot\vec{1}_{\psi_{\mathrm{OA}}} \quad \textbf{(14)}$$

with

$$\vec{1}_{\vec{R}_{\mathrm{OA}}} := \frac{\vec{R}_{\mathrm{OA}}}{\|\vec{R}_{\mathrm{OA}}\|}, \qquad \vec{1}_{\psi_{\mathrm{OA}}} := \frac{\vec{\omega}_{\mathrm{A}}\cdot dt}{\|\vec{\omega}_{\mathrm{A}}\cdot dt\|} \quad \textbf{(15)}$$

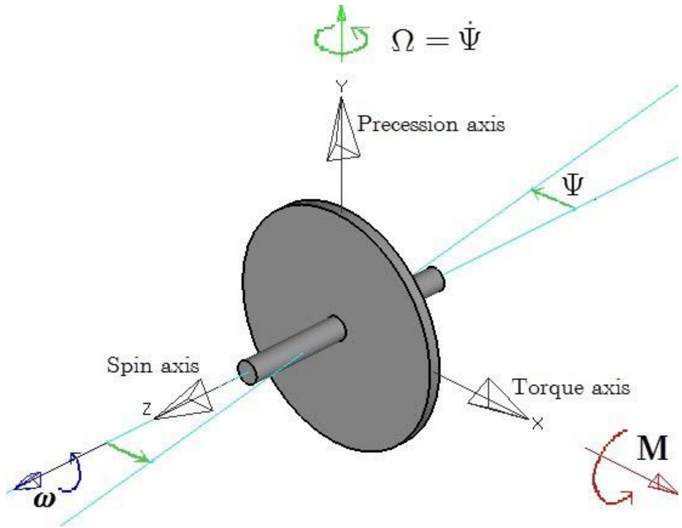

Figure 3 Gyro wheel mechanics:
A right-hand rule is in effect in the relation between spin axis $\vec{z}$, applied torque axis $\vec{x}$, and precession axis $\vec{y}$ [8].

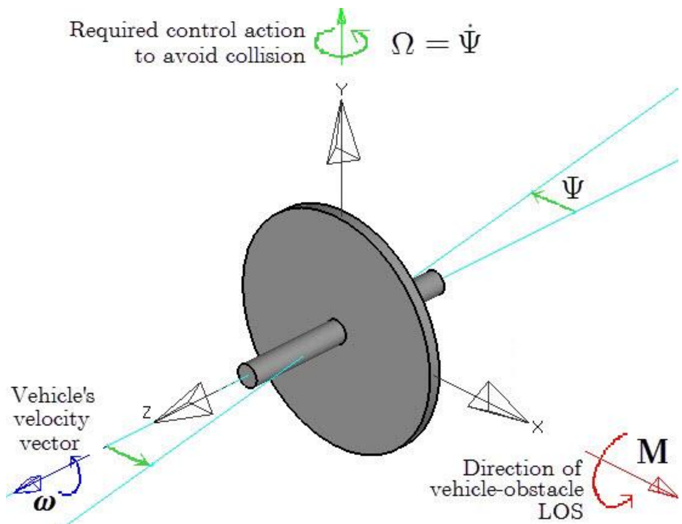

Figure 4 Diagram of CA control by gyroscopic action: Analogy to a spinning gyroscope behavior

The direction of vehicle-obstacle line of sight that is not necessarily perpendicular to vehicle's velocity vector is a more general case than what is shown in Figure 3 and Figure 4.

The CA controller (14) acts based on three information:

1) Relative angular position of an obstacle with respect to vehicle's velocity vector. CA controller (14) will drive the vehicle away from the obstacle by making this angular position higher over time. This contains the angular potential-driven nature of the controller.
2) Magnitude of vehicle's velocity vector. CA controller (14) will acts stronger on faster vehicle.
3) Relative distance to an obstacle. CA controller (14) will acts stronger as relative distance to an obstacle decreases. This contains the translational potential-driven nature of the controller.

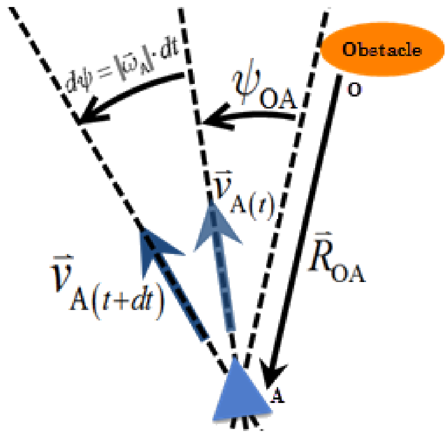

Figure 5 Geometric description of CA control by gyroscopic action

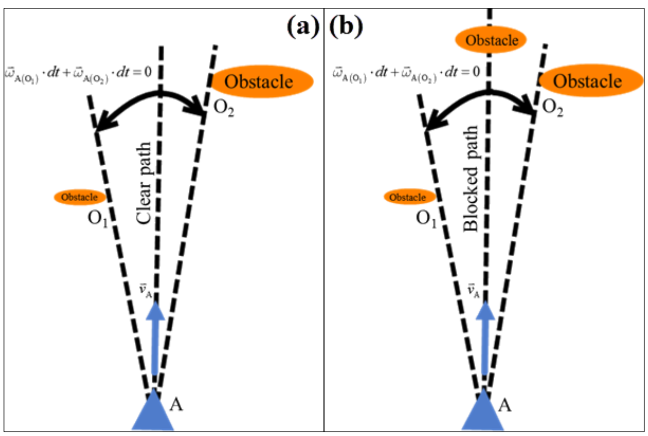

Figure 6 Local minima in angular DoF:
(a) desired path is clear, deadlock situation can be avoided;
(b) desired path is blocked, another control rule is required to find another clear path

As a potential-driven controller, the occurrence of local minima is also possible for CA control system by this approach. But since the controller works only in angular DoF, local minima situation does not necessarily mean the vehicle is trapped by the surrounding obstacles. As long as there is clear path while being in that local minima, collision with nearby obstacles can be avoided (Figure 6).

IV. Collision Avoidance Control by Velocity Obstacle

This particular method was proposed in [7] for a problem of multiple autonomous bodies in a shared environment space. The method is not as direct as the previous two methods. It works by calculating all solutions that results in collision situation, then select one that is in the complementary set of these solutions. Usually, the one closest to the desired state is selected since it will minimize the value of the cost function of overall control system.

In the n-body system, each body (robot's or obstacle's) is assumed to occupy a finite circular (or spherical for 3D) space around a point. The state of each body is described by its motion variables such as position vectors and velocity vectors, which only available to other robots through active observation; such state is called external state. To avoid colliding with any other bodies, each robot (robot A) in this system is task to independently select a new velocity $\vec v_{\text{Anew}}$ for itself such that all robots and obstacles in the environment are guaranteed to be in collision-free situation for at least a finite amount of time τ when they would continue moving at their new velocities without colliding with each other.

The collision-free velocity is selected by setting new velocity that is not in the velocity obstacle set. The velocity obstacle set is formulated as the following.

For any two robots A and B, the velocity obstacle for robot A that is induced by a body B (an obstacle or another robot) for a finite time interval τ, that is, ${}_{\text{Obs}}V_{\text{A|B},\tau}$ is the set of all relative velocities of robot A with respect to body B that will result in collision condition between robot A and body B at some time t within the next time interval τ, hence ${}_{\text{Obs}}V_{\text{A|B},\tau}$ is the velocity obstacle to robot A.

Introducing formulation of an open disc, or sphere of a robot, with radius r and centered at location indicated by $\vec p$, then the personal space D of the robot is a set of all points indicated by $\vec q$ that satisfies (16).

$$D_{(\vec p, r)} = \left\{ \vec q \mid \left\| \vec q - \vec p \right\| < r \right\} \quad \textbf{(16)}$$

Then, velocity set ${}_{\text{Obs}}V_{\text{A|B},\tau}$ is defined as

$${}_{\text{Obs}}V_{\text{A|B},\tau} = \left\{ \vec v_{\text{A}} \mid \exists t \in \left[0, \tau\right] :: \vec v \cdot t \in D_{(\vec p_{\text{B}} - \vec p_{\text{A}}, r_{\text{A}} + r_{\text{B}})} \right\} \quad \textbf{(17)}$$

The velocity set ${}_{\text{Obs}}V_{\text{A|B},\tau}$ can be described geometrically as shown in Figure 7 and Figure 8.

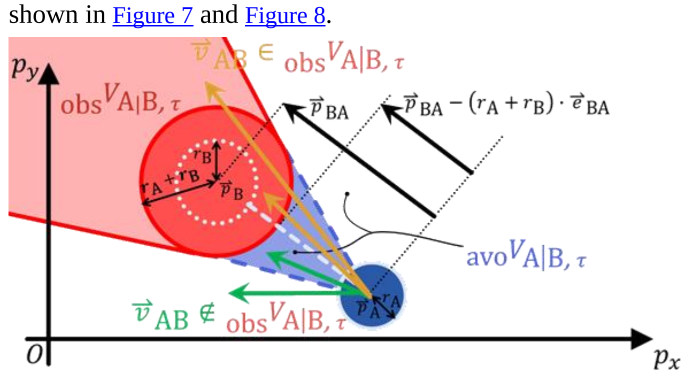

Figure 7 Geometric representation of velocity obstacle to body A in the presence of body B, in position space.

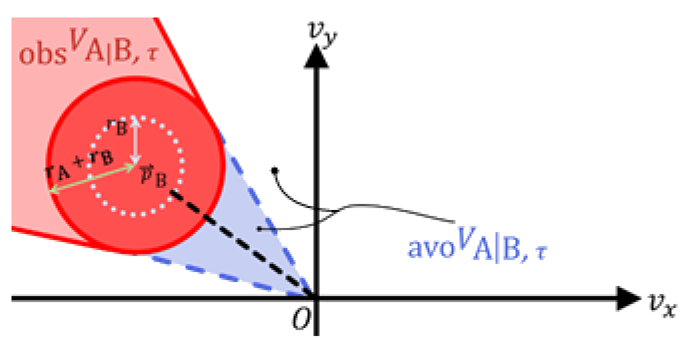

Figure 8 Geometric representation of velocity obstacle to body A in the presence of body B, in velocity space.

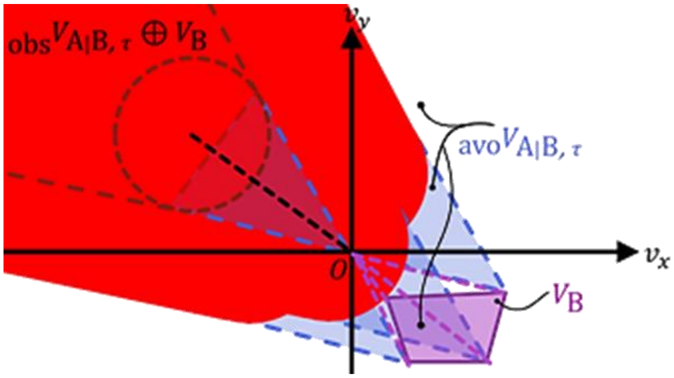

Figure 9 Geometric representation of velocity obstacle to body A in the presence of body B, in velocity space (velocity constraint of body B is considered)

Definition by equation **(17)** implies if $\vec{v}_A - \vec{v}_B \in {}_{Obs}V_{A|B,\tau}$, or equivalently, $\vec{v}_B - \vec{v}_A \in {}_{Obs}V_{B|A,\tau}$, then robot A and robot B will collide within time interval τ if either of them does not change its current velocity vector. It follows that collision avoidance is guaranteed if either one of the two expression in **(18)** is satisfied.

$$\begin{aligned} {}_{CA}\left(\vec{v}_A - \vec{v}_B\right) &\notin {}_{Obs}V_{B|A,\tau} \\ {}_{CA}\left(\vec{v}_B - \vec{v}_A\right) &\notin {}_{Obs}V_{A|B,\tau} \end{aligned} \quad \textbf{(18)}$$

Then, the collision avoiding velocity vector can be formulated as

$$\begin{aligned} {}_{CA}\vec{v}_{A(t)} &\in {}_{CA}V_{A,\tau}, \qquad t_0 \le t < t_0 + \tau \\ {}_{CA}V_{A,\tau} &= {}_{Obs}\bar{V}_{A|B,\tau} \end{aligned} \quad \textbf{(19)}$$

with $\bar{V}$ is the complementary set of V.

If by certain mean, body B's motion constraint is known to robot A, then, we can include it in the collision-free velocity for robot A formulation.

$$\begin{aligned} {}_{CA}\vec{v}_{A(t)} &\in {}_{CA}V_{A|B,\tau}, \qquad && t_0 \le t < t_0 + \tau \\ {}_{CA}V_{A|B,\tau} &= \bar{W}, && W := {}_{Obs}V_{A|B,\tau} \oplus V_B \end{aligned} \quad \textbf{(20)}$$

Operator $\oplus$ denotes the Minkowsky sum, described in **(21)**. Figure 9 illustrates the overall velocity obstacle to robot A considering body B's motion constraint.

$$V_A \oplus V_B = \left\{\vec{a} + \vec{b}\right\}, \quad \vec{a} \in A,\ \vec{b} \in B \quad \textbf{(21)}$$

Velocity obstacle formulation **(20)** can be rewritten for multiple obstacle case in an *n*-body system. For each robot *i* with multiple obstacles (index *j*) present in its vicinity,

$$\left.\begin{aligned} \vec{v}_{i(t)} &\in {}_{CA}V_{i|j,\tau}, \qquad && t_0 \le t < t_0 + \tau \\ {}_{CA}V_{i|j,\tau} &= \bar{W}, && W := \bigcup_{j=1}^{n-1} {}_{Obs}V_{i|j,\tau} \oplus V_j \end{aligned}\right\} \quad \textbf{(22)}$$

$${}_{Obs}V_{i|j,\tau} = \left\{\vec{v}_i \mid \exists t \in \left[0,\tau\right] :: \vec{v}_i \cdot t \in D_{\left(\vec{p}_j - \vec{p}_i,\, r_i + r_j\right)}\right\} \quad \textbf{(23)}$$

CA solution always exists as long as ${}_{CA}V_{i|j,\tau}$ is not empty set.

$${}_{CA}V_{i|j,\tau} \ne \varnothing \quad \textbf{(24)}$$

V. Discussion and Conclusion

It is shown in section 2 and section 3 that potential-driven CA can be applied to nonholonomic system since this approach may also works in the angular channels by defining angular potential field. But, most works, if not all, only make use of the translational aspect of this approach while the angular aspect is being done either by coupling the controller's angular DoF to translational DoF, or by other different approaches. On the other hand, gyroscopic action approach turns out to be a lot more similar with potential-driven CA. In this regard, the gyroscopic approach CA can be seen as the angular aspect of potential-driven CA that stands on equal ground with its translational aspect.

The notable difference between velocity obstacle approach and the potential-driven approach is that it evaluates all 'forbidden' solutions, then, CA solution is obtained by selecting a solution that is not forbidden. Since all solutions (all possible forbidden solutions and their complementary counterparts the CA solutions) are evaluated for a finite time interval τ, it is possible to 'focus' all available vehicle's sensory resources only on the nearest obstacle bodies that fall within that time interval τ, *e:g:*, if the vehicle is to collide with some obstacle body after some τ elapsed, then all other obstacle bodies that are at a farther distance do not matter. Therefore, the presence of many obstacle bodies in the environment does not necessarily put higher load on vehicle's computing resources. Also, since CA solutions are evaluated since the beginning of the control method (as the complementary solution of forbidden solution), this approach is more resistant to deadlock situation as also apparent from its geometric illustration (Figure 8 & Figure 9).

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