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Techniques for Quadcopter Modelling & Design: A Review

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Abstract— Quadcopters modelling research has cease or almost at a halt, compared to its controller counterpart, which has received all the attention and continue to receives more research attention. While controller design seems to be viewed as a way of addressing the inherent problems peculiar to quadcopters by ways of controlling it, it no surprising to see that a lot of previous research failed to address this inherent practical problem. This paper reviews the most effective techniques widely used by researchers for modelling & designing quadcopters only and not it controller. Rotor modelling in conjunction with aerodynamic effect, parametric identifications were reviewed and analysed.

As a demonstration, a quadcopter modelled was developed via voltage manipulation approach for this review, likewise PID controller. Taylor-series first-order approximation was used to linearize the developed non-linear model so that linear PID controller can be used to test the model.

This model the author envisage to discuss with Mathwork Inc. for its incorporated as into the Matlab software as a quadcopter Simulink-block; for which different controllers can be directly interfaced to (with little as required design modification to the helicopter block model or no modification at all). This, the author believes will aid further study of the inherent research problems associated with quadcopter.

Keywords— Dynamic Modelling, Mathematical and Simulink Modelling, System Theory and Identification, PID, Quadcopter.

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I. Introduction

The last decades has witnessed tremendous use of UAV (Unmanned Aerial Vehicles) for academic research, civilian-entertainment, military, etc. due to improvements in the technologies & modelling techniques used for designing the helicopter as well as their associated control systems. State-of-art [3,26,27,37] attributes this increase to the helicopter's versatility and the peculiar low design complexities of quadcopters because of the fact that this class of UAV (i.e. quadcopter) relies on fixed pitch rotors with variations in motor (or rotor) speed for the design of vehicle control system without complex mechanical control linkages. However, these attributes comes at a price because quadcopters are highly under-actuated and as such controlling them becomes a challenge for designers and researchers [17]. Furthermore, this challenge is compounded by the high non-linear dynamics of quadcopters as well as the several uncertainties (modelling, design, environmental, etc.) [13,41]. Quadcopters, like other helicopters, must provide their own damping for hovering, deceleration, stopping, etc., but unlike ground vehicles, their motions experiences very little friction.

The foregoing discussions create some very interesting challenges for the design, modelling and control of quadcopters.

To address these challenges, different approaches have been proposed in different literatures and research publications about altering the conventional shapes used in modeling and designing quadcopters. With the conventional approach, each rotor is mounted per corner equidistant from the centre-of-mass of the helicopter while synchronized rotational speed of all the rotors is key to effective modelling and control of quadcopter [1,37]. There is the gyroscopic effect modelling approach for high RPM motors proposed by state-of-art [24] but this gyroscopic effect will vanish during linearization of the quadcopter model. There is the proposed technique of rotors rearrangements suggested in state-of-art [14] but these techniques will be more complex than the conventional design technique used for modelling and controlling the helicopter with no additional benefit(s) or improvement(s). State-of-art [22] proposed a technique of using different sized rotors with the hypothesis that some of the rotors be fixed at different distances from the quadcopter's origin; investigation of this configuration was deduced to result in the generation of different thrust disturbances & aerodynamic drag and in addition lead to violation of the peculiar symmetry assumptions proposed by different researchers and established quadcopter's literatures. Another proposed design approach reported by [14,22] involves a design-technique in which one of the rotor directions is altered by 90-degrees for axial motion generation only. Investigation results for this technique shows that offers a chance for the system to get fast motion along one axis as well as increases the maneuverability at the expense of additional controller design for stabilization problem only; this further compounds the controller design problems [14,22].

The motivation behind this paper review stems from recent shift in UAV research towards a more payload oriented missions device (i.e. pick and place or mobile manipulation missions) and aggressive flight trajectories. This paradigm will need a more complete dynamic model of the helicopter and as result this paper reviews and organizes the techniques

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associated with quadcopters design & modelling from widely accepted publication & literatures and making inference from these. State-of-art [19,21] demonstrated in their research that for demanding flight trajectories, such as fast forward, descent flight manoeuvres, etc., controller actions diminishes in situations where aerodynamic effects such as ground effects, blade-flapping, etc., are not properly incorporated in the quadcopter's model.

Reviewed techniques herein presented in this paper are all based on the de-facto cross-shaped or plus-shaped configuration due to their successes and versatility in design applications.

II. Modeling and Lineariztion

A. *Surveyed Configuration Types and Analysis of Dynamic Model*

Ghazbi *et al* [30] shows different forms of quadcopter used for various purposes: research, aerial photography, surveillance, etc.

Depending on the helicopter's blade-orientation relative to its body-coordinate, all of these different models can be classified broadly into either of the following configurations: plus-configuration or cross-configuration. The former configuration, employed by DraganFlyer XPro, uses a pair of blades spinning in the same direction, which are placed on x and y coordinates of the body frame. With this configuration it is easier to control the aircraft since each movement (either in the x or y direction) requires a controller to balance only the speed of two blades responsible for that desired direction [5,25]. While the latter configuration employed by most design such as Parrot AR, Curtiss-Wright VZ-7AP, etc., on the contrary uses motion from all rotors at every instant to produce high maneuverability and greater momentum [15]. Either configuration is widely accepted amongst researchers and designers but according to Getsov *et al* [14] & Mahony *et al* [25], the cross-configuration gain more acceptance because of it stability to changing speed of each blade by a small amount, as opposed to changing only two blades.

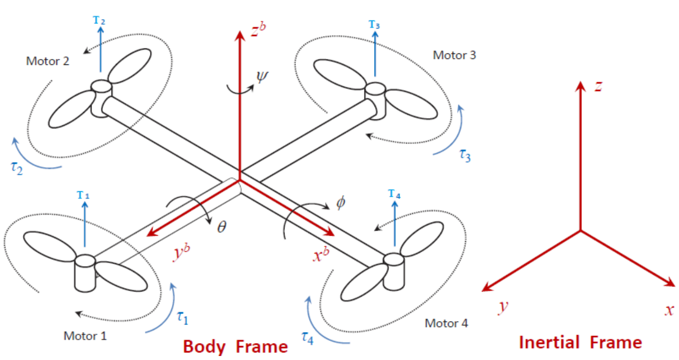

Figure 1 Quadcopter spatial free-body diagram representation; roll, pitch and yaw angles are denoted by ϕ, θ, ψ respectively. Rotors 1 and 3 rotate clockwise while 2 and 4 rotate counter-clockwise as depicted by the arrows. T_1, T_2, T_3 and T_4 are the thrusts generated by the rotors about their centre of rotation. τ_1, τ_2, τ_3 and τ_4 are the torques applied to the quadcopter (counter torques) as a consequence of the spinning of the rotors.

A prerequisite for developing the equations-of-motions for the helicopter is the demonstration of the quadcopter's reference frames. Quadcopters are usually defined in spatial-orientation using a two reference frame systems, which are presented in Figure 1 [1,7,16,37] and defined as follows:

- Body (or Mobile) frame is defined by ground, with gravity pointing in the negative Z-direction. The body-frame vectors describing the helicopter's linear & angular positions are generally represented as: translational velocities $[u, v, w]^{\mathrm{T}}$ and rotational velocities $[p, q, r]^{\mathrm{T}}$.
- Inertia (or Earth) frame is defined by the orientation of the quadcopter with its rotor axes pointing in the positive z-direction and the arms pointing in the X and Y-directions. Within this frame, position $[x, y, z]^{\mathrm{T}}$ and attitude (roll, pitch & yaw) $[\phi, \theta, \psi]^{\mathrm{T}}$ describes its linear & angular positioning.

Combination of these four vectors presented in the body and inertia frame represent the 12-states of the quadcopter.

According to [32] and [39], quadcopters are described dynamically as a highly non-linear helicopter modelled with the following attributes:

- It is a 12-states helicopter (six attitude-states and six position & linear velocity states)
- It possesses 6-DOF (3-translational velocities and 3-rotational velocities)
- It is actuated by four independent rotors.

Inference on this description by various researchers concludes that the resulting helicopter's dynamics is a severely under-actuated and a highly nonlinear helicopter marred with erratic aerodynamic uncertainties (because it control-inputs uses four rotors to control its 6-DOF).

While quadcopters are capable of many forms of movements, most literature and publications focus mostly on the following movements:

- Roll-motion corresponds to quadcopter's rotation about the X_b-axis. It is obtained when $\omega_2 = \omega_4 = \omega_{\text{hover}}$ and ω_1 and ω_3 are changed. For a positive roll, $\omega_1 > \omega_{\text{hover}}$ and $\omega_3 < \omega_{\text{hover}}$. A negative rolling action is produced when we set $\omega_1 < \omega_{\text{hover}}$ and $\omega_3 > \omega_{\text{hover}}$.
- Pitch-motion corresponds to the quadcopter's rotation about the Y_b-axis is obtained when $\omega_1 = \omega_3 = \omega_{\text{hover}}$ and ω_2 and ω_4 are altered. For a positive pitch, $\omega_2 > \omega_{\text{hover}}$ and $\omega_4 < \omega_{\text{hover}}$ while negative pitch action is obtained when $\omega_2 < \omega_{\text{hover}}$ and $\omega_4 > \omega_{\text{hover}}$.
- The yaw motion corresponds to a rotation of the quadcopter about the Z_b-axis. It is produced by the difference in the torque developed by each pair of rotors; with a pair creating a clockwise-torque while the other pair anticlockwise-torque. By varying the angular speed of one pair over the other, the net torque applied to the helicopter generates the yaw motion. A positive yaw action is obtained by setting $(\omega_1 = \omega_3) > \omega_{\text{hover}}$ and $(\omega_2 = \omega_4) <$

ω_{hover}. A negative yaw action is achieved when $(\omega_1 = \omega_3) < \omega_{\text{hover}}$ and $(\omega_2 = \omega_4) > \omega_{\text{hover}}$.

- The VTOL motions (change in the z-frame) is obtained by equally augmenting or diminishing the angular speed of all motors with respect to ωhover and hence the supply voltage. This lead to a vertical upwards (or downwards for decent) force with respect to body-fixed frame which raises or lowers the helicopter.

B. General Model Assumption

The modeling approaches of quadcopters are based on the physics of the system with further partitioning of the system into smaller subsystems for easy analysis, design and modeling [10]. Generally in quadcopter modeling, different assumption have been made by simplification of the mathematical equations needed for the helicopter modeling while still establishing a fairly accurate model as possible. Universal model assumptions from different literatures and research publications are herein presented as under [1],[7,8],[11],[16],[32],[38],[42]:

- The Quadcopter has a perfect symmetrical structure and the inertia matrix about this symmetry is diagonal.
- The physical structure of the Quadcopter is rigid frame equipped with four motors.
- The bearing pressure and the trail of each engine are proportional to the square speed of each motor, which is an approximation very close to the aerodynamic behavior of the helicopter.
- Centre of Gravity (CoG) of the helicopter is fixed about it origin.
- The helicopter is time-invariant and hence there are no mass changes during motion.
- Thrust and drag constants are proportional to the square value of motor's speed.
- All motors on the quadcopter are identical, so a single motor can be analyzed without loss of generality.
- Aerodynamically effects such as blade-flapping and non-zero free stream velocity are usually ignored.

C. Survey of Different Modeled Equations

Different literatures and research papers have been published in which different mathematical techniques were used for modeling the helicopter from first principle approximation, ranging from quaternion formulation demonstrated in [23] through "Superposition-formulation" used in [5] to the Newton-Euler technique; amongst these, the two universally used techniques in deriving the helicopter's model equations:

- Newton-Euler– This method is based on Newton's second law equations on rigid body [4,38]. Please refer to included references for more details.
- Euler-Lagrange– This method is based on energy and kinematics, with equations derived from Newton's second law using speed transformation matrix [37,39]. Please see references for more details.

Both techniques generate the same results and are preferred due to their efficacy.

From this derivation techniques, the mathematical modeled equation-of-motions surveyed from various academic literatures and publications falls into one of the following popular categories:

1) Angular Orientation Approach

This approach uses Newton-Euler technique to develop one of quadcopter's famous model equations for the 6-DOF. Using this approach the 6-DOF were deduced as follows:

- The equation for the roll subsystem was deduced as

$$\ddot{\phi} = \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_r}{I_x}\dot{\theta}\Omega + \frac{l}{I_x}U_2 \tag{1}$$

- The equation for the pitch subsystem was deduced as

$$\ddot{\theta} = \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) - \frac{J_r}{I_y}\dot{\phi}\Omega + \frac{l}{I_y}U_3 \tag{2}$$

- The equation for the yaw subsystem was deduced as

$$\ddot{\psi} = \dot{\phi}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) + \frac{1}{I_z}U_4 \tag{3}$$

- the height, X-motion and Y-motion are as

$$\left.\begin{aligned} \ddot{z} &= -g + (\cos\phi\cos\theta)\frac{U_1}{m} \\ \ddot{x} &= (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\frac{U_1}{m} \\ \ddot{y} &= (\cos\phi\sin\theta\sin\psi + \sin\phi\cos\psi)\frac{U_1}{m} \end{aligned}\right\} \tag{4}$$

Abbasi & Mahjoob [1] and Zulu & Samuel [42] use this approach in their modeling.

2) Force-Moment Approach

Force-Moment Approach is also based on the Newton-Euler techniques. This approach uses relationship between force and moment balance to develop the helicopter equation-of-motions, which is presented as

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{pmatrix} {}^{E}_{B}R\left\{\begin{bmatrix} 0 \\ 0 \\ \sum F_i \end{bmatrix} - \begin{bmatrix} K_1\dot{x} \\ K_2\dot{y} \\ K_3\dot{z} \end{bmatrix}\right\}\frac{1}{m} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \\ (F_1 - F_2 - F_3 + F_4 + K_4\dot{\phi})\frac{l}{I_x} \\ (-F_1 - F_2 + F_3 + F_4 + K_5\dot{\theta})\frac{l}{I_y} \\ (CF_1 - CF_2 + CF_3 - CF_4 + K_6\dot{\psi})\frac{1}{I_z} \end{pmatrix} \tag{5}$$

F_i is the individual force produced by each rotor, C is a constant relating moment to force, K_i is the coefficient of the drag opposing the helicopter motion for $i = 1, 2, \cdots, 6$. I_x, I_y, I_z are the helicopter's modelling moment of inertia with respect to x_B, y_B, z_B axes respectively. In this approach, for convenience, drag coefficients are assumed to be zero since drags are negligible at low speed [9–11].

3) Voltage-based Approach

This approach derived the quadcopter's equation-on-motion using Newton-Euler technique based on the input voltages supplied to the helicopter rotors; this is summarized under [7,16].

- The equation for the roll subsystem was deduced as

$$\ddot{\phi} = \frac{2\rho Al}{I_{xx}}\left(\frac{f\eta K_t}{K_q}\right)^2 (V_2^2 - V_4^2) \quad \textbf{(6)}$$

- The equation for the pitch subsystem was deduced as

$$\ddot{\theta} = \frac{2\rho Al}{I_{yy}}\left(\frac{f\eta K_t}{K_q}\right)^2 (V_3^2 - V_1^2) \quad \textbf{(7)}$$

- The equation for the yaw subsystem was deduced as

$$\ddot{\psi} = \frac{J}{I_{zz}}(\dot{\Omega}_1 + \dot{\Omega}_3 - \dot{\Omega}_2 - \dot{\Omega}_4) + \frac{D}{I_{zz}}(\Omega_1^2 + \Omega_3^2 - \Omega_2^2 - \Omega_4^2) \quad \textbf{(8)}$$

- The equation for the altitude subsystem was deduced as

$$\ddot{z} = \frac{2\rho A}{m}\left(\frac{f\eta K_t}{K_q}\right)^2 (V_1^2 + V_2^2 + V_3^2 + V_4^2)\cos\phi\cos\theta - g \quad \textbf{(9)}$$

- The equation for the X-axis subsystem was deduced as

$$\ddot{x} = \frac{2\rho A}{m}\left(\frac{f\eta K_t}{K_q}\right)^2 (V_1^2 + V_2^2 + V_3^2 + V_4^2)(\cos\phi\cos\theta\cos\psi + \sin\phi\sin\psi) \quad \textbf{(10)}$$

- The equation for the Y-axis subsystem was deduced as

$$\ddot{y} = \frac{2\rho A}{m}\left(\frac{f\eta K_t}{K_q}\right)^2 (V_1^2 + V_2^2 + V_3^2 + V_4^2)(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) \quad \textbf{(11)}$$

D. Model linearization

Mathematical models and equations-of-motions derived via the above aforementioned techniques are nonlinear in nature and according to Chaturvedi [10], this nonlinearity makes it difficult for classical controllers (like PID) to effectively be used for controlling quadcopters without transforming it from the time-domain to the Laplace or s-domain. In this regards, Zhang *et al* [41] mentioned in their work that traditionally, first principle assumptions and measurements parameters often find usage in developing linear quadcopter models from their nonlinear models. Linearization of this time-invariant non-linear model is a necessary condition in obtaining a more classical controller, friendly linear-model [37].

One of the most popular linearization methods for quadcopter is the small disturbance theory employed by Bolandi *et al* [4] and Zambrano [40]. Another widely used method is the Taylor-series transformation; Bousbaine *et al* [7], Poyi [16], and da Costa [32] use first-order Taylor-series to linearize their non-linearized state-space model equation from

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \quad \textbf{(12)}$$

$$\Downarrow$$

$$\dot{\boldsymbol{x}} = \frac{\partial f}{\partial \boldsymbol{x}}(\boldsymbol{x} - \boldsymbol{x}_0) + \frac{\partial f}{\partial \boldsymbol{u}}(\boldsymbol{u} - \boldsymbol{u}_0) \quad \textbf{(13)}$$

where the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are defined as the Jacobian of a set of non-linear equations about the initial conditions. The state variables $\boldsymbol{x}$ is given as

$$\boldsymbol{x} = [u, v, w, p, q, r, x, y, z, \phi, \theta, \psi]^{\mathrm{T}} \quad \textbf{(14)}$$

Although gyroscopic effects are incorporated in most modelling equation, linearization of the helicopter usually eliminates this effect. According to Bolandi *et al* [4] this elimination is attributed to the high RPM of the motors.

Another linearization technique linearizes the state-space model equation at hovering equilibrium point.

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x}) \cdot \boldsymbol{u} \quad \textbf{(15)}$$

where $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$ and $\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u}$.

According to Deif *et al* [38], the linear model after performing Taylor-series approximation at hovering equilibrium and eliminating high order terms yields

$$\dot{\phi} = p, \quad \dot{\theta} = q, \quad \dot{\psi} = r \quad \textbf{(16)}$$

$$\dot{u} = -g\theta, \quad \dot{v} = g\phi, \quad \dot{w} = g - \frac{F}{m} \quad \textbf{(17)}$$

$$I_{xx}\dot{p} = \tau_\phi, \quad I_{yy}\dot{q} = \tau_\theta, \quad I_{zz}\dot{r} = \tau_\psi \quad \textbf{(18)}$$

Mellinger *et al* [28] expands on this state-space linearization by conducting linearization for three additional equilibrium points beside the hovering equilibrium point performed by Zhang *et al* [41]. These points are:

- vertical movement at fixed velocity
- horizontal movement at fixed roll angle
- horizontal movement at fixed pitch angle

E. Rotor modelling and associated aerodynamic

While different class of motor exists for use in modelling quadcopter's rotor, most research literatures and publications model this by using brushless DC motors due to their high torque and little friction. Using the voltage equation-of-motion modelling perspectives, the rotor is modelled as: [7,16]

$$v = J\Omega\frac{R}{k_e} + k_e\Omega + K_q R\Omega^2 \quad \textbf{(19)}$$

Equivalent angular perspective representation of this model is given as

$$\frac{d\Omega}{dx} = -\frac{k_e K_q}{JR}\Omega - \frac{\tau_d}{J} + \frac{K_q}{JR}V \quad \textbf{(20)}$$

Since quadcopter's rotor RPM varies linearly between the square of the angular velocity against thrust & drag, Tanveer et al [36] and Dief & Yoshida [37] proposed a technique which focuses on altering the rotor's RPM value to generate the corresponding thrust and drag via system identification analysis.

Equally important in rotor modelling are two prime aerodynamics effects: Rotor blade flapping and ground effect. Flapping of the rotor blades is a phenomenon which induces forces in the x-y plane of the helicopter' rotor; these forces are significantly important while modelling quadcopters since they help determine the helicopter's natural stability by ways of how high gain control can easily be achieved against the effects induced by this forces [24,40]. Most quadcopters controllers often can handle small disturbances naturally but struggle to counter the effect arising from rotor blade flapping [21]. Regarding blade-flapping, Huang *et al* [21] suggest that for effective modeling of the quadcopter and its controller to counter blade flapping effects, the design should incorporate feed-forward compensation to annul the resultant forces and moments generated by rotor flapping.

Another prime factor which is usually omitted from quadcopter modelling is "Ground-Effect", a phenomenon of counter reaction by the ground when the helicopter flies close to it (say half it rotor's diameter). This effect causes flight imbalance by way of pushing the helicopter away from the ground [5,30]. A proposed mathematical model of ground-effect was postulated by Johnson [23] and presented by Deif *et al* [38], this is shown herein as:

$$\frac{T}{T_\infty} = \frac{1}{1-\left(\frac{R}{4z}\right)^2} \tag{21}$$

where in this context T is the rotor's thrust against the ground; T_∞ the thrust produced outside ground-effect, R is the rotor's radius and z the vertical distance of the helicopter from the ground.

While ground-effect is worth accounting for in modelling of the helicopter, Zambrano [40] demonstrated that this effect is negligible if the rotors distances from the ground is more than twice its radius, i.e. $\frac{z}{R} > 2$.

III. Rotor Control Inputs Modeling

The Quadcopter is a 6-DOF MIMO system defined by four control-inputs to its rotors, responsible for controlling its 12-output-states (six altitude control states and six position & linear velocity states). This control-inputs for controlling the rotor by ways of manipulating - it angular velocities, it torques, force-moment balance and supplied rotor voltages – generally in all reviewed literatures and publications employed the same notion for representing them: U1 is associated with vertical input movement, U2 associated with roll input movement, U3 with pitch input movement and U4 with yaw input motion [1,41]. These control inputs falls into one the followings categories:

A. Force-moment Control-Input

Employed by Chaturvedi [10] is presented as:

$$U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\sum_{i=1}^{4} F_i \\ \frac{1}{I_{xx}}(F_1 - F_2 - F_3 + F_4) \\ \frac{1}{I_{yy}}(-F_1 - F_2 + F_3 + F_4) \\ \frac{C}{I_{zz}}\sum_{i=1}^{4}(-1)^{i+1}F_i \end{bmatrix} \tag{22}$$

B. Angular velocity

Employed by [1] & [41], angular velocity is presented herein as under:

$$U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \tag{23}$$

$$U_2 = lb(-\Omega_2^2 + \Omega_4^2) \tag{24}$$

$$U_3 = lb(-\Omega_1^2 + \Omega_3^2) \tag{25}$$

$$U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \tag{26}$$

C. Thrust-length control input approach

Da Costa [32] in his thesis design presented his length and thrust approach, presented as follows:

- Vertical thrust (sum of all thrust)

$$U_1 = l(T_1 + T_2 + T_3 + T_4) \tag{27}$$

- Rolling moment (thrust difference between two opposite motors)

$$U_2 = l(T_4 - T_2) \tag{28}$$

- Pitching moment (thrust difference between two opposite motors)

$$U_3 = l(T_1 - T_3) \tag{29}$$

- Yawing moment (algebraic sum of all torques)

$$U_4 = l(T_1 - T_2 + T_3 - T_4) \tag{30}$$

D.

Bouabdallah [5] solves the following equations to generate the necessary control inputs for their helicopter model.

$$\ddot{z} = g - \frac{U_1}{m}\cos\phi\cos\theta \tag{31}$$

$$\ddot{\phi} = \frac{d}{I_{xx}}U_2 - \frac{Jm}{I_{xx}}\dot{\theta}\Omega + \frac{I_{yy}-I_{zz}}{I_{xx}}\dot{\theta}\dot{\psi} \tag{32}$$

$$\ddot{\theta} = \frac{d}{I_{yy}}U_3 - \frac{Jm}{I_{yy}}\dot{\phi}\Omega + \frac{I_{zz}-I_{xx}}{I_{yy}}\dot{\phi}\dot{\psi} \tag{33}$$

$$\ddot{\psi} = \frac{d}{I_{zz}}U_4 + \frac{I_{xx}-I_{yy}}{I_{zz}}\dot{\phi}\dot{\theta} \tag{34}$$

E. Voltage control input approach

This approach is employed by Bousbaine *et al* [7] and Poyi [16].

- Vertical thrust (z-axis)

$$U_1 = V_1 + V_2 + V_3 + V_4 \tag{35}$$

- Rolling moment (y-axis)

$$U_2 = V_4 - V_2 \tag{36}$$

- Pitching moment (x-axis)

$$U_3 = V_1 - V_3 \tag{37}$$

- Yawing moment

$$U_4 = V_1 - V_2 + V_3 - V_4 \tag{38}$$

F. Rotor Saturation & Dead Zone

Although often omitted from most literatures and research publication, rotor saturation is an important aspect of quadcopter design and modelling. Rotor saturation helps determine the magnitude and behavior of rotors' responses to control inputs which it can or cannot handle effectively [37]. As a result, control action on the helicopter is usually limited by this motor constraints; given a situation where the control inputs to the motors are over their maximum limits, this might lead to failure of the motors. Hence incorporating or fathoming saturation into rotors' modeling will effectively lead to the development of a more superior model whose boundary constraints are well established. Furthermore, saturation is usually considered non-linear in nature and it greatly affects the quadcopter response time to control signal [20,33], *e.g.*, it expands the settling time more than normal.

Also, while using brushless DC motors for quadcopter modelling, the effect of "Dead-Zone" should be incorporated in the rotor modelling. Dead-zone is that a region of operation in brushless DC motors where the motor generates no rotational motor and torque after receiving signal for rotation. According to state-of-art [37], this region is not constant and varies with motors and their speed controllers.

IV. Our Matlab-Simulink Quadcopter Model

The voltage equation-of-motion approach developed by [7,16] was used to develop our quadcopter Matlab-Simulink model of the helicopter and its associated control-input splitter subsystem. Our developed Simulink model is presented in Figure 2–Figure 6 while our model design parameters are shown in Table 1.

Table 1 Model design parameter

Symbol	Parameters	Value	Units
m	Quadcopter total mass	0.65	kg
l	Length of Quadcopter arm	0.19	m
I_{xx}	Rotational inertia along x-axis	0.0075	kg·m^2
I_{yy}	Rotational inertia along y-axis	0.0075	kg·m^2
I_{zz}	Rotational inertia along z-axis	0.013	kg·m^2
R_p	Rotor blade length	0.16	m
f	Rotor blade figure-of-merit	0.5	
J_r	Rotor's inertia	6.0e-5	kg·m^2
R	Motor's resistance	0.6	ohm
K_e	Rotor's speed constant	0.0015	volts·s·rad^{-1}
K_q	Rotor's torque constant	0.0056	N·m·A^{-1}
η	Rotor's efficiency	0.75	%
K_t	Torque constant	0.01	N·s^{-2}
g	Acceleration due to gravity	9.81	m·s^{-2}
D	Drag coefficient	7.50e-7	
ρ	Air density	1.1	kg·m^2

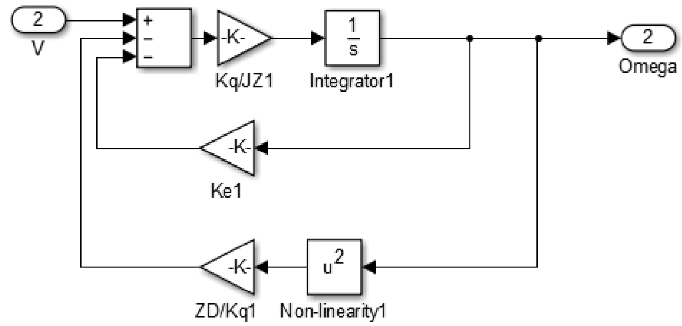

Figure 2 Motor Model

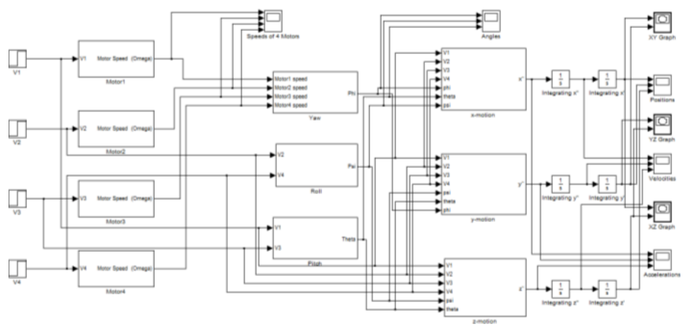

Figure 3 Simulink Quadcopter Model

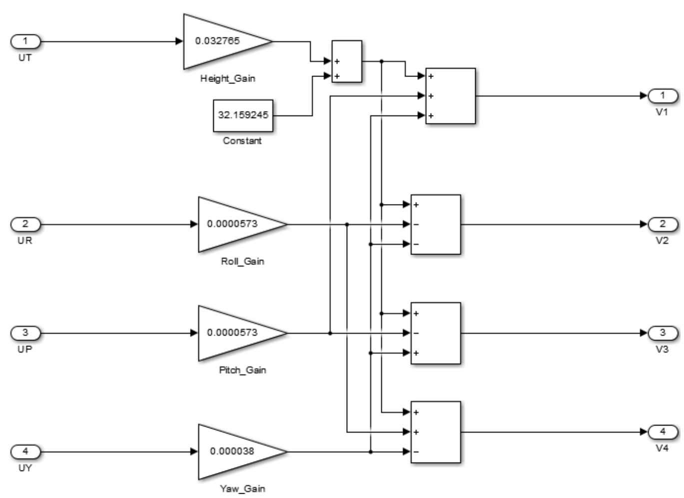

Figure 4 Simulink Voltage Splitter subsystem

Solver used for our Simulink model is ode45 at an input sampling time of zero; simulation runtime was 50sec. Our control input expressed in matrix-form herein presented, was used to develop the Simulink model presented in Figure 5.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0.25 & 0 & 0.5 & 0.25 \\ 0.25 & -0.5 & 0 & -0.25 \\ 0.25 & 0 & -0.5 & 0.25 \\ 0.25 & 0.5 & 0 & -0.25 \end{bmatrix} \begin{bmatrix} V_1 + V_2 + V_3 + V_4 \\ V_4 - V_2 \\ V_1 - V_3 \\ V_1 - V_2 + V_3 - V_4 \end{bmatrix} \quad \text{(39)}$$

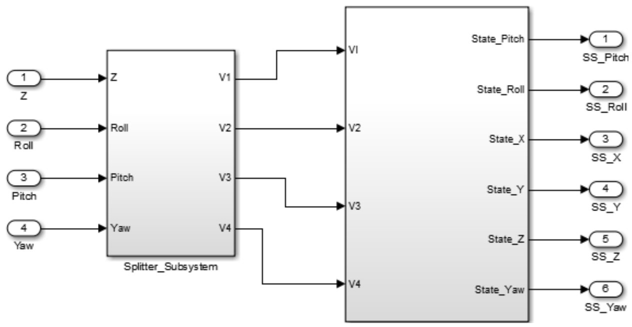

Figure 5 Interfacing Quadcopter and Voltage splitter units

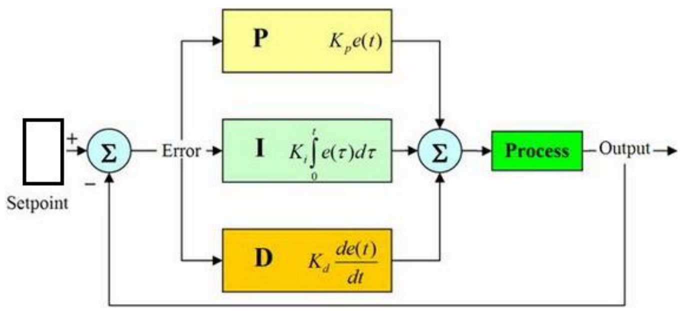

Figure 6 Classical PID control-algorithm [2]

A. Linearization of our model

First-order Taylor-Series approximation was used to linearize the non-linear quadcopter model developed via the mathematical equations. The linearization follows the four point model used by Zambrano [40], with salient aspects of the linearized equation expressed as follows:

Linearization at hovering implies that the quadcopter's x-y body-frame is parallel to the x-y inertia-frame; at the same time the vehicle's roll, pitch and yaw angles are all equal to or approximately zero. The derivation for the control voltage used for achieving these conditions via height equation-of-motion is as follows:

$$\ddot{z} = \frac{2\rho A}{m}\left(\frac{f\eta K_t}{K_q}\right)^2 (V_1^2 + V_2^2 + V_3^2 + V_4^2)\cos\phi\cos\theta - g \quad \textbf{(9)}$$

with

$$\phi = \theta = \psi = \dot{\phi} = \dot{\theta} = \dot{\psi} = \ddot{\phi} = \ddot{\theta} = \ddot{\psi} = 0$$

$$\Omega = \Omega_{\mathrm{h}}$$

$$\dot{\Omega} = \ddot{\Omega} = 0$$

$$\dot{x} = \ddot{x} = \dot{y} = \ddot{y} = \dot{z} = \ddot{z} = 0$$

Substituting the constants values from Table 1 and the conditions listed above, the resulting equation becomes:

$$0 = \left(\frac{2.2 \cdot 0.08042}{0.65}\right)\left(\frac{0.005 \cdot 0.75}{0.0056}\right)^2 (V_1^2 + V_2^2 + V_3^2 + V_4^2)\cos 0 \cos 0 - 9.81$$

$$0 = (0.27219)(0.44842)(V_1^2 + V_2^2 + V_3^2 + V_4^2)\cos 0 \cos 0 - 9.81$$

$$0 = (0.12206)(V_1^2 + V_2^2 + V_3^2 + V_4^2)\cos 0 \cos 0 - 9.81$$

$$V_{\mathrm{h}} = V_1 = V_2 = V_3 = V_4$$

$$0 = (0.12206)(4V_h^2)\cos 0 \cos 0 - 9.81$$

$$V_{\mathrm{h}} = \sqrt{20.0933}$$

$$V_{\mathrm{h}} = 4.48 \text{ volts}$$

We assumed that all the rotors acts exactly in similar manner and the quadcopter's design symmetric; hence their control voltage at operating point are thought to be equal.

B. PID

Classical PID-algorithm depicted in Figure 6 was used as the control platform for testing our developed quadcopter model. Generally, a PID controller is a feedback control-loop mechanism that continuously calculates an error-value difference between a desired set-point and a measured variable and applying controlling action by compensating for the error-difference; the controller attempts to minimize the error over time by adjusting the control variable (such as voltage supplied to the Quadcopter) to a new value determined by a weighted sum of its control-law given as:

$$u(t) = K_{\mathrm{P}} e(t) + K_{\mathrm{I}} \int_0^t e(\tau)d\tau + K_{\mathrm{D}} \frac{de(t)}{dt} \quad \textbf{(40)}$$

K_{P}, K_{I}, and K_{D} are all non-negative and denotes the coefficients for the proportional, integral and derivative terms respectively [2].

V. Analysis and Discussions

Qualitative analysis from surveyed literatures and publications shows that the Newton-Euler approach to developing the helicopter's mathematical model is highly favored amongst researchers, hobbyist and technologist. Even though other approaches produce the same results, say in spite of the compact formulation and generalization demonstrated while using Euler-Lagrange approach [39]. The choice of Newton-Euler approach usage lies in its historical development which renders it easy to use & apply. This was demonstrated in our model which was developed via the Newton-Euler approach.

Due to the available abundance of well-studied tools for linear system designs, linear model of quadcopters are widely used even though non-linear models provides great insight into the behaviour of the helicopter. Although the simplistic approach of modeling and controlling quadcopter suit most basic operations such as hovering and slow velocity flight due to neglecting rotor's aerodynamic effects & the likes [40]. For bellicose manoeuvring such as fast forward flight, fast-climb, fast-VTOL, etc. which are significantly affected by the quadcopter aerodynamic, requires new modelling paradigms. These paradigms necessitate for a research shift in the development of more elaborate mathematical equation models to deal with the shortcomings of traditional simplistic modelling approach [20,26,40]. Huang *et al* [21] demonstrated the inefficiencies of existing modelling and control techniques to accurately account for high-speed trajectory tracking when model uncertainties exist and aerodynamic modelling effects are neglected [21].

System identification is an alternate and powerful technique for deriving the helicopter model directly from test data yet this technique has gain little attention from researchers [18].

Findings from our model shows that controller actions on the quadcopter model works pretty fine but as the helicopter ascends to greater heights, short overshoot was observed as well as increase response time for other subsystems to reference track the desired signal. Hence we find that each subsystem developed via the equation-of-motions presented by researchers is partially independent of each other, to some extent.

While it is universally agreed that the quadcopter is a 12-state helicopter, controller design (either linear or non-linear controllers) focus only on six controllable states (x,y,z,ϕ,θ,ψ) due to the lack of full controllability and observability for all the 12 states. According the work by Balas [43], who reached conclusion that quadcopter's model are not fully controllable for all 12 states (but fully observable for all states); reduction of the total states of the helicopter's model into the following six prime states of interest to researchers, produces a fully controllable and observable state equation model.

$$\dot{\boldsymbol{x}} = \frac{d}{dx}[x \quad y \quad z \quad \phi \quad \theta \quad \psi]^{\mathrm{T}} \tag{41}$$

$$\boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du} = [x \quad y \quad z \quad \phi \quad \theta \quad \psi]^{\mathrm{T}} \tag{42}$$

Hence, justifying the reason why most quadcopter research papers and publications focus on these six controllable and observable states.

Another research gap is the lack research model for the effect of ceiling on quadcopter during indoor flight, a phenomenon known as "Ceiling-Effect". During indoor flight, Ceiling-Effects pulls the quadcopter towards the ceiling as it approaches the ceiling, which can eventually leads to crashing of the helicopter. Modeling of this effect has not really been given proper attention by quadcopter research community.

VI. Conclusions

This paper presented a review on the generally accepted approaches for modeling quadcopters. While the popular techniques used for modeling the equation-of-motions of the helicopter generate similar results, Newton-Euler approach appeases the senses of researchers and hobbyist.

Linearized model, derived from non-linear model via first-principle Taylor-Series approximation, was used in our model in conjunction with PID-controller to demonstrate our voltage modeling approach; this produces excellent result for basic movement like VTOL, rolling, pitching, etc. However, aggressive maneuver shows the inefficiencies of the linear model, say slow response time for instance. Hence, full non-linear modeling will be required for adequate modeling of the helicopter to suit the aforementioned research direction of quadcopters. The author believes that model is the foundation for proper controller design; hence the incorporation of uncertainty and aerodynamic effects modeling in the modeling of the quadcopter, will effectively improves quadcopter design and hence controller actions.

Generally, quadcopter models irrespective of modeling approaches are characteristically an unstable, multivariable, non-linear dynamic system marred with complex rotor's aerodynamic effects and under-actuation. While most research focuses on developing controllers to tackle discrepancies between it actual system and mathematical-model, the author without bias nor interest of conflicts beliefs modeling tactics for the helicopter need a fresh approach or improvements; most publication and literatures admit and hide under the umbrella that the under-actuation of quadcopters makes it difficult to develop a more model of the helicopter. As quadcopter research shift to more aggressive researches with payload picking-up and delivery, the need for an intricate mathematical model will incorporate a full aerodynamic-effect spectrum and the helicopter under-actuation arises and need properly be addressed.

VII. Future Work

CFD analysis of the quadcopter model is required to understand aerodynamic effects the helicopter as each subsystem is not mutually exclusive of the other.

Ackowledgement

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GLOSSARY

τ_d	load-torque (N·m)
ω	angular-velocity (rad·s^{-1})
$\dot{x}$	rate of change of a particular state
η	motor’s efficiency (%)
ψ	yaw-angle (rad)
ϕ	roll-angle (rad)
θ	pitch-angle (rad)
Ω	angular-velocity of motor (rad·s^{-1})
A	state-transition matrix
B	input-matrix
C	measurement-matrix
D	input feed-forward matrix
b	thrust
d	drag-coefficient
F	force (N)
g	acceleration due to Gravity (m·s^{-2})
i	motor-current (Amps)
I	identity-matrix
I_{xx}, I_{yy}, I_{zz}	rotational inertia about the x, y, and z-axis respectively (N·m·s^{-2})
J	motor’s moment of inertia (kg·m^2)
k_e	motor’s back-EMF constant (N·m/A)
k_q	motor torque-constant (Nm/A)
l	distance from center of the chassis-frame and the motor (m)
m	mass of quadcopter (kg)
R	motor-resistance (ohms)
T	thrust (N)
U	Control input to rotor
u	State-space control signal
V	input voltage to quadcopter motor (volts)

x	quadcopter state
X	x-axis of the quadcopter
y	output-measurement
Y	y-axis of the quadcopter
Z	z-axis of the quadcopter
CFD	computational fluid dynamics
DOF	degree-of-freedom
MIMO	multi-input multi-output
RPM	revolution per minute

SUPLEMENTARY FILES

Name	**Size**	**Type**
981-3536-1-SP.pdf	469 kB	Portable Document Format File
981-3430-1-SP.slx	35 kB	Simulink Model File