

A Linear Dynamic Model Approximation of a Flapping-type Tidal Turbine

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Abstract—In this paper, a linear dynamic model of flapping-type tidal turbine (FTT) is proposed. Firstly, a nonlinear dynamic model is derived to predict the equation of motion of the system. Then, the nonlinear dynamic model was analyzed and validated using simulation and experimental data. Secondly, a linear dynamic model is derived from the nonlinear dynamic model using approximation method. To validate this method, the proposed linear dynamic model is analyzed and validated using simulation and experimental results. This dynamic model is also compared to another linearization method. The result showed that the proposed linear dynamic model could represent the characteristic of the system as in the nonlinear dynamic model.

Keywords—Unmanned underwater vehicle, robust control, extended H-infinity, parametric uncertainty, Multiobjectives Differential Evolution.

I. INTRODUCTION

THE research development of tidal turbine is rapidly increasing these days. Though the most developed methods such as horizontal axis turbine and vertical axis turbine are still the leading edge of the tidal turbines technology, companies and researchers have been trying to develop the new method of extracting the energy from the ocean [1]. Of course, the new developed method should at least overcome the drawbacks of the most developed methods.

The new developed method that is discussed here is the flapping-type method. This method has been actively investigated and developed by companies and researchers. For instance, the Stingray system in UK by Engineering Business Limited [2] and Pulse 100 by Pulse Tidal Limited [3] are some of the FTT systems that are developed by the companies. The collaboration between KIOST and Konkuk University have produced the small scale model of FTT [4,5,6,7]. The reliability of the systems regarding to the flow and the frequency oscillation of the systems have been tested and verified [6,7,8]. Naval University and KIOST has also developed an FTT with tandem hydroplane system [9,10].

In this paper, a linear dynamic model of FTT is proposed. The linear dynamic is derived from the nonlinear dynamic

model using approximation method. The linear dynamic model is analyzed and validated using simulation and experimental result

II. NONLINEAR DYNAMIC MODEL

The dynamic model of FTT system can be derived using the diagram shown in **Figure 1**. The pitch axis of hydroplane is located at the center of hydrodynamic forces which is at the quarter of the chord length (0.25c). The pitching moment at hydrodynamic center is minimum and does not vary with lift coefficient at any angle of attack. The hydroplane is a rigid wing with the shape of airfoil NACA0012.

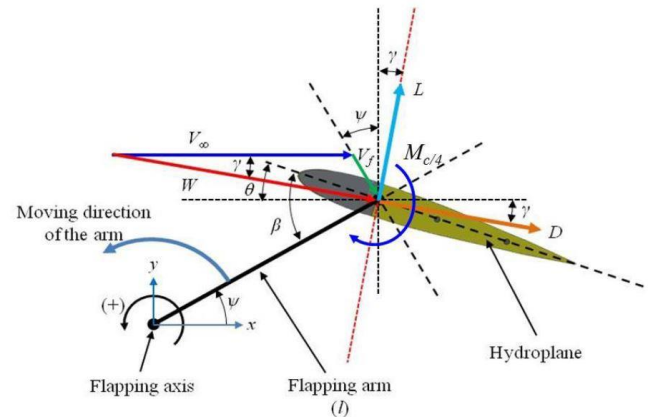


Figure 1 Force diagram acting on a hydroplane

To derive the valid dynamic model, the pitch angle of hydroplane (θ), flapping arm angle (ψ), the far field inflow speed (V_∞) and the relative flow velocity (W) of the water towards the hydroplane are used. The relative flow velocity of the water towards the hydroplane is expressed as follows:

$$W = \sqrt{(V_\infty + l\dot{\psi} \sin \psi)^2 + l^2 \dot{\psi}^2 \cos^2 \psi}, \quad (1)$$

where l and $\dot{\psi}$ are the length of the flapping arm and the angular flapping speed, respectively.

As the nose of hydroplane is up, the effective angle of attack is described as follows:

$$\alpha = \theta + \gamma, \quad (2.a)$$

where γ is the deflection angle of the flow.

$$\gamma = \tan^{-1} \left(\frac{W_y}{W_x} \right). \quad (2.b)$$

Due to the relative flow velocity W , hydrodynamic forces and pitching moment that acting on the hydroplane are determined as follow:

$$L = \frac{1}{2} \rho C_L S [(V_\infty + l\dot{\psi} \sin \psi)^2 + l^2 \dot{\psi}^2 \cos^2 \psi], \quad (3.a)$$

$$D = \frac{1}{2} \rho C_D S [(V_\infty + l\dot{\psi} \sin \psi)^2 + l^2 \dot{\psi}^2 \cos^2 \psi], \quad (3.b)$$

$$M_{c/4} = \frac{1}{2} \rho C_M S c [(V_\infty + l\dot{\psi} \sin \psi)^2 + l^2 \dot{\psi}^2 \cos^2 \psi], \quad (3.c)$$

where C_L , C_D , C_M , ρ , and S are lift, drag, pitching moment coefficients, and the reference area of the hydroplane, respectively. The lift, drag, and pitching moment coefficients are function of the effective angle of attack α .

The horizontal and vertical forces in x and y directions are defined as follow:

$$F_x = -L \sin \gamma + D \cos \gamma, \quad (4.a)$$

$$F_y = L \cos \gamma + D \sin \gamma. \quad (4.b)$$

Moment due to the hydrodynamic force about the flapping axis is described as:

$$\sum M = -F_x (l \sin \psi) + F_y (l \cos \psi) - M_{c/4}. \quad (5)$$

In order to measure the power that can be extracted from the flapping oscillation, holding torque τ is constantly applied at the output shaft of the flapping axis. Thus, the equation of motion of the system is expressed as follows:

$$(I + I_g) \ddot{\psi} = \sum M + \tau \quad (6)$$

By combining Equations 3, 4, 5 and 6, the dynamic model of the system becomes:

$$(I + I_g) \ddot{\psi} + b\dot{\psi} - \frac{1}{2} \rho S l X (Y + Z) + M_{c/4} - \tau = 0, \quad (7)$$

$$X = (V_\infty + l\dot{\psi} \sin \psi)^2 + l^2 \dot{\psi}^2 \cos^2 \psi,$$

$$Y = (C_L \cos \gamma + C_D \sin \gamma) \cos \psi,$$

$$Z = (C_L \sin \gamma - C_D \cos \gamma) \sin \psi.$$

Here, b is the damping coefficient of the whole system, I is the mass moment of inertia of the hydroplane including

flapping arm around the flapping axis, and I_g is the equivalent mass moment of inertia of the gear box around the flapping axis. The damping coefficient of the whole system, b , is determined through simulation such that the measured and predicted responses of the rigid wing be in good agreement. The holding torque is defined as follows:

$$\tau = \begin{cases} T, \dot{\psi} \leq 0 \\ -T, \dot{\psi} > 0 \end{cases} \quad (8)$$

where T is the magnitude of the applied holding torque.

The lift, drag, and pitching moment coefficients are obtained from the characteristic of NACA0012 according to Sandia Laboratory report. The graphic presentation of these coefficients is shown in **Figure 2**.

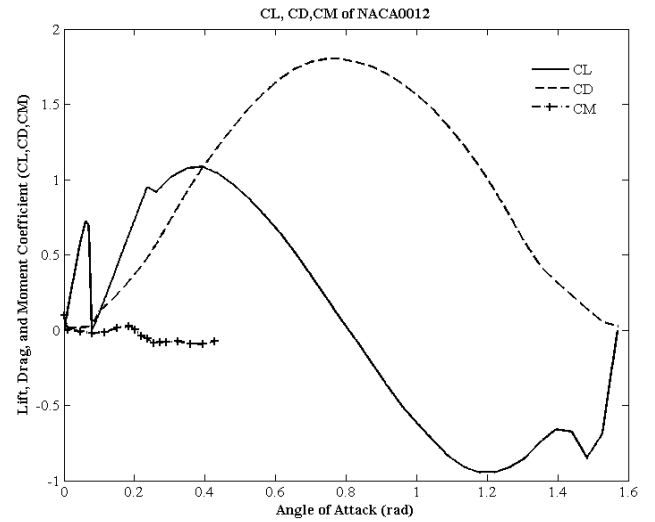


Figure 2 Lift, drag, and moment coefficients of NACA0012

The dynamic model of FTT system can be coded using MATLAB[®] Simulink as shown in **Figure 3**. Lift, drag and pitching moment coefficients are included using lookup table. The input is the hydroplane pitch angle θ and the output is the flapping arm angle ψ .

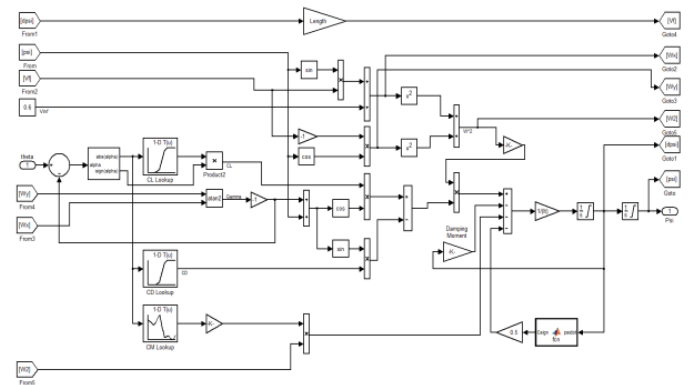


Figure 3 Simulation block of dynamic model of FTT system using MATLAB[®] Simulink

III. LINEAR DYNAMIC MODEL

In NACA0012, the values of lift, drag, and pitching moment coefficients are nonlinear. The linear model approximation of the system can be approached by assuming the linear expression of the lift, drag, and the pitching moment coefficients.

The hydroplane NACA0012 has unique lift, drag, and pitching moment coefficients where the value of the lift coefficients changes faster than the value of the drag coefficients as the angle of attack increases. On the other hand, the values of pitching moment coefficient changes slower than the other two coefficients as the angle of attack increases.

The lift, drag, and pitching moment coefficients are the functions of the effective angle of attacks. The effective angle of attack is defined as Equation 2.a. and the deflection angle is defined as Equation 2.b. Hence, the effective angle of attack is actually small. The approximation of the lift, drag, and pitching moment coefficients are expressed as follow:

$$\begin{aligned} C_L &= k_1 \sin 4(\theta + \gamma), \\ C_D &= k_2 \sin 2(\theta + \gamma), \\ C_M &= k_3(\theta + \gamma), \end{aligned} \quad (9)$$

where, k_1 , k_2 , and k_3 are the peak amplitudes of the approximated functions. With the small angle of flapping arm and small angle of attack of hydroplane, Eq. 7 can be simplified as follows:

$$\begin{aligned} (I + I_g)\ddot{\psi} + b\dot{\psi} &= \frac{1}{2}\rho S l R - M_{c/4}, \\ R &= (V_\infty)^2 \left(4k_1(\theta + \gamma) + 2k_2(\theta + \gamma)(\gamma - \psi) \right), \\ M_{c/4} &= \frac{1}{2}\rho S c (V_\infty)^2 k_3(\theta + \gamma). \end{aligned} \quad (10)$$

Since flapping arm angle and the effective angle of attack are small, the deflection angle of the flow is also small, the multiplication of these terms is negligible compared to $4k_1(\theta + \gamma)$. Hence, the linear dynamic model of the system can be expressed as:

$$(I + I_g)\ddot{\psi} + b\dot{\psi} = \frac{1}{2}\rho S (V_\infty)^2 (4k_1 l(\theta + \gamma) - ck_3(\theta + \gamma)) \quad (11.a)$$

By following the assumption at the small angle of flapping arm, the deflection angle of the flow can be expressed as follows:

$$\gamma \approx \frac{-l\dot{\psi}}{V_\infty} \quad (11.b)$$

Thus, the differential equation of linear dynamic model of the system can be obtained as:

$$\ddot{\psi} + \frac{(bV_\infty + Ql(4k_1l - ck_3))}{(I + I_g)V_\infty} \dot{\psi} - \frac{Q(4k_1l - ck_3)\theta}{(I + I_g)} = 0, \quad (11.c)$$

where, $Q = \frac{1}{2}\rho S (V_\infty)^2$. Thus, the transfer function of the system can be obtained as:

$$G(s) = \frac{\psi(s)}{\theta(s)} = \frac{Q(4k_1l - ck_3)}{(I + I_g)s^2 + \frac{(bV_\infty + Ql(4k_1l - ck_3))}{V_\infty}s} \quad (11.d)$$

By using the nominal parameters in TABLE I, the approximated $k_3 = -0.171$ from Figure 2, and the $k_1 = 6.8$, the linear dynamic model of the system can be obtained.

The transfer function between the flapping arm angle and the hydroplane pitch angle for FTT is obtained as:

$$G(s) = \frac{\psi(s)}{\theta(s)} = \frac{96.57}{s^2 + 71.16s} \quad (12)$$

IV. ANALYSIS AND VALIDATION MODEL

The nonlinear dynamic model is validated using simulation and data experiment of FTT. The nominal parameters of the dynamic model are shown in TABLE I.

TABLE I NOMINAL PARAMETERS FOR FTT

Parameters	Nominal Value
c (m)	0.3
ρ (kg/m ³)	997.13
$(I + I_g)$ (kg.m ²)	2.0040
S (m ²)	0.174
b (kg.m ² /s)	16.5
V_∞ (m/s)	0.6
l (m)	0.375

The effect of damping coefficient is investigated through simulation. A sinusoidal signal with the amplitude 30 degrees is given as the pitch angle input for the nonlinear model. The arm angle response is investigated by changing the parameters as 50%, 100%, and 200% of the nominal values. The arm angle responses of nonlinear model for various damping coefficients are shown in Figure 4. It shows that the arm angle responses follow the sinusoidal motion as the input pitch angle with different phase. The oscillation of arm angle is not centered at zero arm angle. The response of arm angle with nominal damping coefficient has better oscillation shape than those with the other damping coefficients.

Then, the effect of total mass of inertia is investigated by changing the values as 50%, 100%, and 200% of nominal mass of inertia. The arm angle response of nonlinear model for various mass of inertias is shown in Figure 5. At the nominal mass of inertia, the response of arm angle is oscillating in sinusoidal motion and not centered at the zero arm angle.

The effect of frequency is investigated by giving the sinusoidal pitch angle with several frequencies. The arm angle

response of nonlinear model for various frequencies is shown in **Figure 6**. The response of arm angle is not centered at zero angle of arm angle. The amplitude of arm angle response is decreased as the frequency signal for pitch angle increases.

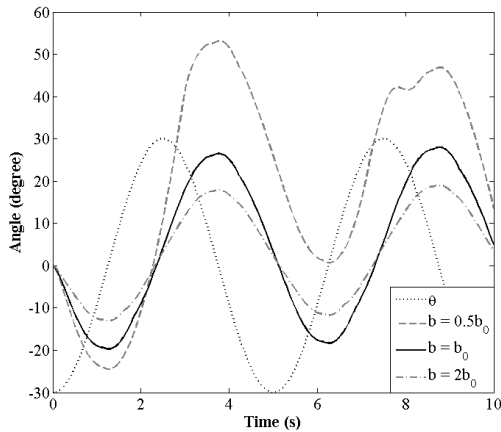


Figure 4 Arm angle response of nonlinear model with various damping coefficients for FFT

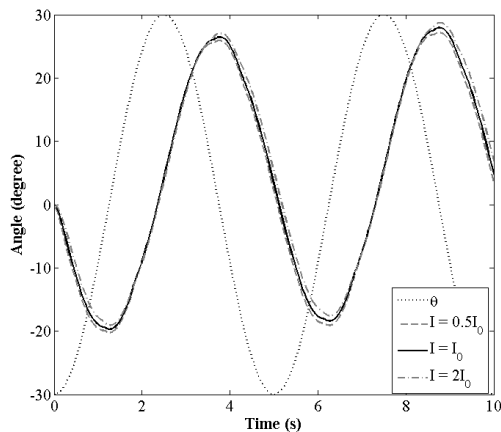


Figure 5 Arm angle response of nonlinear model with various mass of inertias for FFT

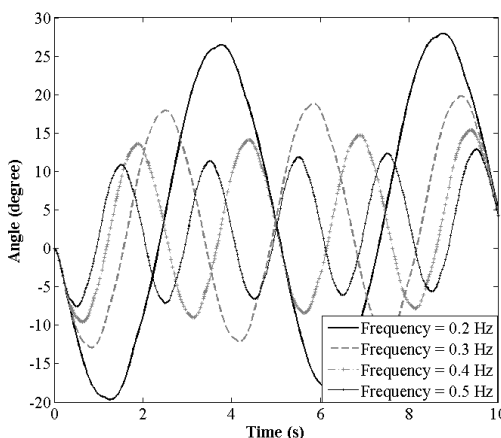


Figure 6 Arm angle responses of nonlinear model with various input frequencies for FFT

After checking the validity of nonlinear dynamic model using simulation with the given sinusoidal input for pitch angle, the validity of nonlinear dynamic model is investigated using experiment result. Here, the input for pitch angle is using the pitch angle input that is used for the experiment of FFT. The effect of damping coefficient is investigated by changing the values as 50%, 100%, and 200% of the nominal damping coefficient. The arm angle responses of nonlinear dynamic model of FFT for various damping coefficients are shown in **Figure 7**.

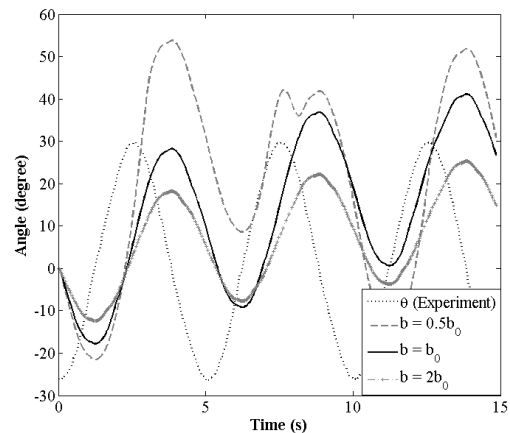


Figure 7 Arm angle responses of nonlinear model with various damping coefficients

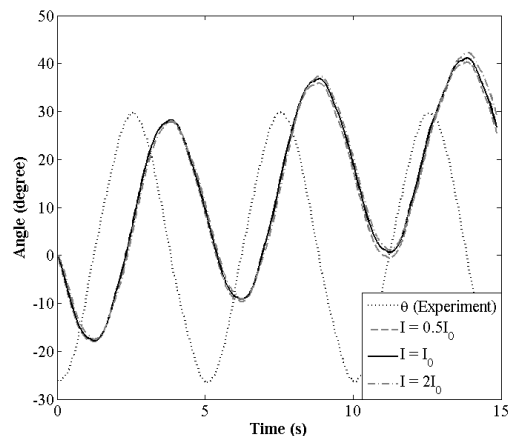


Figure 8 Arm angle response of nonlinear model with various mass of inertias

The result shows that at the nominal damping coefficient, the arm angle response is oscillating in sinusoidal motion as the input pitch angle with different phase. The amplitude is oscillated from negative to positive amplitude as the time increases. Similar response also happened with higher damping coefficient than the nominal damping coefficient. This is due to the fact that the pitch angle input is not centered at zero pitch angle.

Then, the effect of the mass of inertia is investigated by changing the values as 50%, 100%, and 200% of the nominal

mass of inertia. The responses of arm angle for this investigation are shown in **Figure 8**. The responses for all parameters are similar. It means that the nominal parameter that is used for this nonlinear dynamic model is valid.

Based on the analysis of nonlinear dynamic model by changing parameters, it is confirmed that the nominal parameters that are used for FTT are valid. Then, the validity of nonlinear dynamic model for towing-tank-scale can be verified by comparing the arm angle responses of real experiment and nonlinear model. The same input that is used for the real experiment is applied to the nonlinear dynamic model. The comparison of the responses is shown in the **Figure 9**. It shows that the responses are following the sinusoidal motion as the input pitch angle. Even though amplitude of the arm angle response of nonlinear model is oscillated from negative to positive amplitude as the time increases, it is observed that the responses have the same phase.

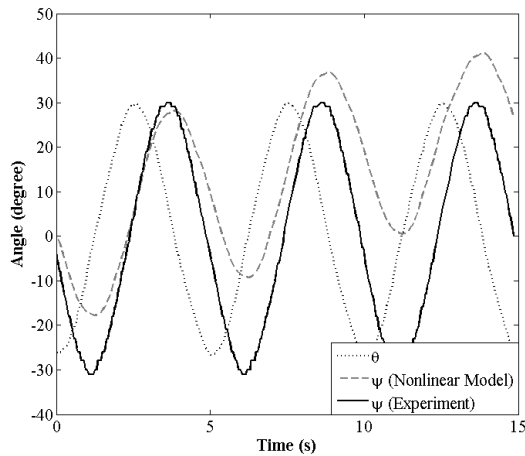


Figure 9 Comparison of arm angle responses for FTT

A linear model of dynamic system also can be obtained using software tool from MATLAB® Simulink. In **Figure 3**, the nonlinear model of the system has been built using MATLAB® Simulink. By linearization method, the transfer function of the system between the flapping arm angle and the hydroplane pitch angle can be obtained.

By using the parameters in **TABLE I**, the transfer function of the system for FTT can be expressed as:

$$G(s) = \frac{\psi(s)}{\theta(s)} = \frac{96.82}{s^2 + 71.37s + 0.1019} \quad (13)$$

Comparing the Equation 12 and Equation 13, the result shows that the transfer function of the system using linear model approximation neglects the forces caused by the drag force and pitching moment

The linear model is validated using simulation of given sinusoidal pitch angle and also the pitch angle of towing-tank-scale experiment result. The arm angle response for linear model of FTT with the given sinusoidal pitch angle is shown in **Figure 12**. It shows that the arm angle shape follows the given pitch angle input. The arm angle response is oscillated

at zero angle of arm angle and has 90 degrees of phase difference to the input pitch angle.

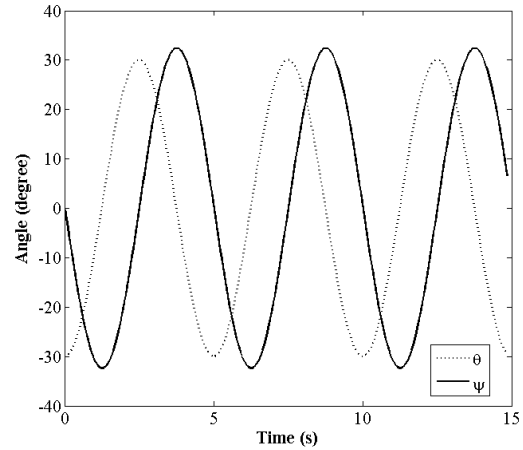


Figure 12 Arm angle response of linear dynamic model for FTT using pitch angle as a sinusoidal signal

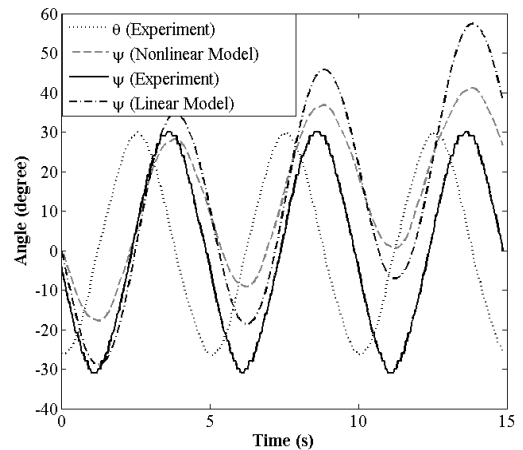


Figure 13 Arm angle responses of FTT

Using experimental data of FTT, the arm angle responses are investigated. The pitch angle data that is used as a pitch angle input for FTT experiment is applied as the input to nonlinear model and linear model. The responses of these dynamic models are compared to the response of arm angle in the experiment. The comparison of these responses is shown in **Figure 13**. It shows that all the responses are oscillating in sinusoidal motion as the pitch angle input. The responses are oscillated in different phase with the pitch angle input, at 90 degree difference. The oscillations of arm angle response of nonlinear model and linear model are not centered at zero arm angle. The fact that the pitch angle input is not centered at zero input angle could cause this problem. Nevertheless, the shape of arm angle responses is following the pitch angle input well.

V. CONCLUSION

The nonlinear and linear dynamic model of the flapping-type turbine (FTT) has been proposed. The derived linear dynamic model has been validated and verified through simulation and experimental data result. The result showed that the derived linear dynamic model could represent the nonlinear

dynamic of the system. The linear dynamic is also verified by comparison of the linear dynamic model using linearization using software tool MATLAB® Simulink.

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