Admissible Model Matching Fault Tolerant Control based on LPV Fault Representation

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***Abstract*—In this paper, an approach to design an Admissible Model Matching (AMM) Fault Tolerant Control (FTC) based on Linear Parameter Varying (LPV) fault representation is proposed. The main contribution of this approach is to consider parametric faults as a scheduling variable in the LPV fault representation of the system that allows the controller reconfiguration on-line. The proposed strategy is an active technique that requires the fault to be detected, isolated and estimated by a FDI scheme. In case the fault estimation is not available, a passive strategy based on a single AMM FTC controller could be designed. The formulation of AMM is based on the set of admissible behaviors that are characterized by means of Linear Matrix Inequalities (LMIs). The LMIs allow to locate the poles of the close-loop system inside a region of the unit circle and the fault accommodation can be formulated in terms of several LMIs. The effectiveness and performance of proposed approach have been illustrated in simulation considering a thermal hydraulic system.**

***Index Terms*—Fault Tolerant Control, Linear Parameter Varying, Admissible Model Matching, Linear Matrix Inequality.**

I. Introduction

Fault Tolerant Control (FTC) has been consolidated as an important research topic in the control applications during last years [1], [2]. The objective of an FTC approach is to maintain desirable closed-loop performance or an acceptable degradation and preserve stability conditions in the presence of component and/or instrument faults. Accommodation capability of a control system depends on many factors such as severity of fault, the robustness of the nominal system and mechanisms that introduce redundancy in sensors and/or actuators. Generally speaking, FTC systems can be categorized into two main groups: *active* and *passive*. The *passive FTC* techniques are control laws that take into account the fault appearance as a disturbance. Passive FTC technique is designed with the consideration of a set of presumed faults modes. The resulting control system performance tends to be conservative. It also has the limitation to deal with unanticipated faults. In [3], among many others, a complete description of passive FTC approach can be found. On the other hand, the *active FTC* techniques are based on adapting the control law using the information given by the Fault Detection and Isolation (FDI) block [2]. With this information, some automatic adjustments are done trying to reach the control objectives. Active FTC is characterized by on-line FDI scheme and an automatic control reconfiguration mechanism. Two main potential advantages of Active FTC are: 1) the ability to deal with previously unknown faults with explicit Fault Diagnosis and Controller Reconfiguration; and 2) the possibility to achieve the optimal performance. However, the price to pay for these nice features is that the overall system becomes more complicated [4].

Fault accommodation has been addressed in the literature considering many different control objectives and using many different solution techniques. The interested reader can see [5], [6] for a recent review. In model matching approaches, the goal of the fault accommodation is defined in terms of similarity between the closed-loop system and nominal system (optimal behavior). In the *Pseudo-Inverse Method* (PIM) [7], a *model matching* formulation provides a solution that minimizes a distance between the closed-loop accommodated and nominal systems. In particular cases, an *exact model matching* can be obtained, but in the general case the optimality of the obtained solution does not guarantee stability. As alternative, the *Admissible Model Matching* (AMM) approach was proposed in [8] and later extended in [9], [10]. The main idea is to accommodate the controller such that it is guaranteed that the system closed-loop behavior is in the set of admissible behaviors. In the original AMM formulation, the set of admissible behaviors is defined in terms of several inequality constraints relating the closed-loop system matrix coefficients [9].

This paper proposes a reformulation of the AMM method using pole placement constraints in LMI regions [11]. This formulation allows a more versatile, straightforward and systematic characterization of control specifications characterized by Linear Matrix Inequalities (LMIs). This approach allows the controller accommodation problem to be formulated as a LMI-feasibility problem that can be solved numerically using efficient optimization algorithms due to its convex property [12]. The proposed formulation allows to address FTC under parametric faults by solving a LMI-constrained optimization problem.

A solution to represent the parametric faults in linear systems is to use Linear Parameter Varying (LPV) techniques (see [13], [14], among others and recently surveyed by [15]). The idea of this approach is to represent a system as an interpolation of simple (usually affine) local models according to the real-time variation of system dynamics. In this paper, parametric faults are considered as scheduling variables of the LPV fault representation of the system [16]. For affine LPV representation, the interpolation techniques present a good approach to get a polytopic structure. This structure is a set of linear models scheduled by a convex function [17].

This paper considers the previous ideas to develop an AMM

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FTC approach for LTI plants subject to parametric faults that allows to specify the set of admissible faults. The controller is able to tolerate the parametric faults with an admissible performance degradation using pole placement in LMI regions. In case that the fault is considered as a scheduling variable, the faulty plant can be considered as a LPV fault representation. Then, an active FTC strategy can be designed using LPV control theory that requires the fault to be detected, isolated and estimated by the FDI scheme and the controller to be redesigned on-line accordingly. On the other hand, in case that the fault effect could not be estimated because of the unavailability of the FDI system, a passive FTC strategy can be used alternatively. The fault effect can be considered as parametric uncertainty in terms of several LMIs constraints.

The remainder of the paper is organized as follows: Section 2 presents the principle of AMM. Section 3 describes the proposed AMM approach based on the LPV fault representation of the plant, that allows to specify the set of admissible faults to be tolerated while preserving the desired control specifications. The controller is designed using pole placement in LMI regions. Section 4 presents a characterization of the recoverability of the proposed FTC approach based on the LPV fault representation. Finally, Section 5 demonstrates the effectiveness and performance of AMM FTC approach using a thermal hydraulic system.

II. Admissible Model Matching Principle

A. Revision of Admissible Model Matching FTC

In order to recall the principle of Admissible Model Matching proposed in [8], consider the following LTI system:

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the system state, $u(k)\in\mathbb{R}^m$ is the control input, $A\in\mathbb{R}^{n\times n}$ and $B\in\mathbb{R}^{n\times m}$ are constant matrices. A classical state feedback control law is considered:

$$u(k) = -Kx(k) \tag{2}$$

In the model matching problem, (1) and (2) result in the closed-loop behavior that follows the reference model:

$$x(k+1) = (A-BK)x(k) = Mx(k) \tag{3}$$

where M is chosen to be stable.

The main idea of AMM FTC is that instead of looking for a controller that provides an exact (or best) matching to a given single behavior after the fault appearance, a family of closed-loop behaviors that present an acceptable performance is specified [8]. This family is described by $\mathscr{M}$ and corresponds to a set of the matrices such that any solution (3) of satisfying:

$$x(k+1) = Mx(k), \quad M \in \mathscr{M} \tag{4}$$

has an acceptable dynamic behavior. The set of reference closed-loop models $\mathscr{M}$ is defined off-line by the designer.

Definition 1. Admissibility: *The triple (A,B,K) is admissible if and only if it satisfies the condition (4) and can be defined as:*

$$\mathscr{M}_a = \{(A,B,K) : \Phi_{\mathscr{M}}(A,B,K) \le 0\} \tag{5}$$

where $\Phi_{\mathscr{M}}(A,B,K)$ are the set of constraints that guarantee the condition (4) is satisfied.

The set $\mathscr{M}_a$ contains the set of admissible values (A,B,K) such that $(A-BK)\in\mathscr{M}$ can be achieved with the control law (2).

When this approach is applied to FTC, the nominal behavior is characterized by a given pair of matrices (A_n,B_n) while the post-fault behavior is characterized by a different pair (A_f,B_f). For the nominal system operation, a state gain feedback controller K_n that satisfies some nominal control specifications is available such that

$$x(k+1) = (A_n - B_nK_n)x(k) = M^*x(k) \tag{6}$$

where M^* is known as the reference model.

Then when a given fault appears such that the system matrices change to (A_f,B_f), the goal of the fault accommodation is to find a feedback gain K_f that provides an admissible closed-loop behavior:

$$A_f - B_fK_f \in \mathscr{M} \tag{7}$$

Figure 1 summarizes graphically the operating principle of fault accommodation using the AMM approach.

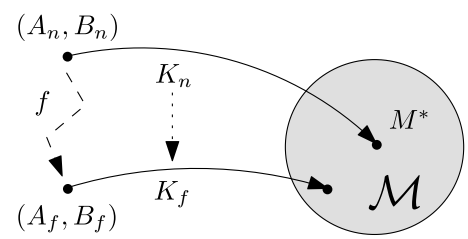

Fig. 1. Fault accommodation using AMM.

In [9], the set of admissible behaviors $\mathscr{M}_a$ is specified by a set of d inequality constraints:

$$\mathscr{M}_a = \left\{M : \Phi(m_{ij},\ i=1,\ldots,n,\ j=1,\ldots,n) \le 0\right\} \tag{8}$$

where $m_{ij},\ i=1,\ldots,n,\ j=1,\ldots,n$ are the entries of matrix M, $\Phi : R^{n\times n} \to R^d$ is a given vector function, and the constraints are written $\Phi(M)$ for short. It is proved that the solution can then be found by solving the constrained optimization problem:

$$\min_M J(M) \tag{9}$$

subject to: $\Phi(M) \le 0$ with:

$$J(M) = \sum_{i=1}^{n} (a_f^i - m^i)^T (I - B_fB_f^+)(a_f^i - m^i) \tag{10}$$

where a_f^i and m^i are respectively the i^{th} columns of A_f and M. In the case that the solution M_f satisfies $J(M_f)=0$, the fault is recoverable and the new feedback gain can be calculated:

$$K_f = B_f^+(A_f - M_f) \tag{11}$$

where B_f^+ is the left pseudo-inverse of B_f, i.e. the matrix such that $B_f^+B_f = I$.

However, it must be noticed that the original formulation (9) has several limitations. First, the set of inequalities (8) associated to a given control performance and a given degradation limit is difficult to be obtained. Some ad-hoc procedures are suggested in [9], [10] but not a systematic one. Second, some specifications are represented by a set of non-linear equations (see the illustrative example in [9]) that leads to the formulation of the fault accommodation as a non-convex optimization problem. These limitations will be overcome in this paper with the new AMM approach proposed.

The AMM approach can also be extended to the tracking problem by adding by adding an integrator in order to eliminate steady-state errors as proposed in [18]. The use of integral control eliminates the need to catalog nominal values or to reset the control. Rather, the integral term can be thought of as constantly calculating the value of the control required at the set point to cause the error to go to zero. To accomplish the design of the feedback gains for the integral and the original state vector, an augmented model is proposed by [19]. The augmented model can be determined with the state $x_I(k)$ and the integral error, $e(k)=y(k)-r(k)$. The discretized integral is implemented as a summation of all past values of $e(k)$, which results in the difference equation:

$$x_I(k+1)=x_I(k)+e(k)=x_I(k)+Cx(k)-r(k) \tag{12}$$

where $r(k)\in\mathbb{R}^{n_r}$ and considering that the output equation associated to (1) is $y(k)=Cx(k)$. Following [19], the augmented model can be expressed as:

$$\begin{aligned}\begin{bmatrix} x_I(k+1) \\ x(k+1)\end{bmatrix} &= \begin{bmatrix} I & C \\ 0 & A_f\end{bmatrix}\begin{bmatrix} x_I(k) \\ x(k)\end{bmatrix}+\begin{bmatrix} 0 \\ B_f\end{bmatrix}u(k)-\begin{bmatrix} I \\ 0\end{bmatrix}r(k) \\ \tilde{x}(k+1) &= \bar{A}\tilde{x}(k)+\bar{B}u(k)-\bar{E}r(k)\end{aligned} \tag{13}$$

where $\tilde{x}(k)\in\mathbb{R}^{n_x+n_r}$ is the augmented state vector and $r(k)\in\mathbb{R}_m$ is a reference vector.

Then, the control law is:

$$\begin{aligned} u(k) &= -[K_I \quad K]\begin{bmatrix} x_I(k) \\ x(k)\end{bmatrix}+\bar{G}r(k) \\ &= -\bar{K}\tilde{x}(k)+\bar{G}r(k)\end{aligned} \tag{14}$$

Thus, the closed-loop system (13) that is designed with the control law (14) is:

$$\tilde{x}(k+1) = \left(\bar{A}-\bar{B}\bar{K}\right)\tilde{x}(k)+\left(\bar{B}\bar{G}-\bar{E}\right)r(k) \tag{15}$$

such that the closed loop behavior follows the reference model

$$\tilde{x}(k+1)=M^{*}\tilde{x}(k)+N^{*}r(k), \tag{16}$$

where $x(k)\in\mathbb{R}_n$ is the system state, $u(k)\in\mathbb{R}_m$ is the control vector and the pair M^*, N^* is given. M^* is chosen to be stable and N^* must satisfy the following condition [19]:

$$CN_x=I \tag{17}$$

Thus, the design of AMM with tracking specifications can be solved using the separation principle: first, the design of feedback control (M^*) is obtained using, for example, pole placement. Then, the reference input is introduced through the control law (15) such that the closed-loop satisfies (16) by including the tracking specification (N^*).

B. Recoverability

Let us denote $\mathscr{F}$ as all possible faults affecting the system (1). According to [9], the FTC specification is defined through the following set.

Definition 2. Set of faults to be tolerated: *Given the system* (1), *the set of fault models that must be tolerated by the controller can be specified as follows:*

$$\left\{\left(A_f,B_f\right),f\in F\subseteq\mathscr{F}\right\}. \tag{18}$$

recalling that the pair (A_f,B_f) denotes the change of system matrices due to the fault f.

For each (A_f,B_f) we are interested in finding an appropriate K_f, where appropriate means that the triple $\left(A_f,B_f,K_f\right)$ is admissible. Thus, the following definitions can be introduced:

Definition 3. Recoverability: *The system (A_f,B_f) affected by a fault is recoverable using the control law* (2) *if and only if the set:*

$$\mathscr{K}(A_f,B_f)=\left\{K:\Phi_{\mathscr{M}}(A_f,B_f,K)\leq 0\right\} \tag{19}$$

is not empty. That means that exist a gain K such that $A-BK\in\mathscr{M}$.

Remark 1. Note that the set $\mathscr{K}(A_f,B_f)$ may contain more than one element and therefore a decision rule to select a unique feedback law in this set must be provided.

Definition 4. Recoverable faults: *The set $\mathscr{R}(\mathscr{M}_a)$ of all* recoverable faults *is:*

$$\mathscr{R}(\mathscr{M}_a)=\left\{\left(A_f,B_f\right),f\in\mathscr{F}:\mathscr{K}(A_f,B_f)\neq\varnothing\right\} \tag{20}$$

and the FTC specification given by Definition 2 can be met if and only if:

$$\left\{\left(A_f,B_f\right),f\in F\subseteq\mathscr{R}(\mathscr{M}_a)\right\} \tag{21}$$

III. AMM FTC using LPV Gain-Scheduling Theory

A. LPV Fault Representation

In this paper, the AMM approach is extended to consider LTI systems with parametric faults represented as an LPV model where the faults are the scheduling variables. Consider the system (13) and its LPV representation using the fault f as the scheduling variable:

$$\tilde{x}(k+1) = \bar{A}(f)\tilde{x}(k)+\bar{B}(f)u(k)-\bar{E}r(k), \tag{22}$$

Note that when $f=0$ corresponds to the fault-free case while $f\neq 0$ the faulty case. Let assume that (22) vary affinely in a polytope with the fault [13]. In particular, the state-space matrices range in a polytope of matrices defined as the convex hull of a finite number of matrices N $(N=2^{n_f})$ where n_f is the number of faults. Each polytope vertex corresponds to a particular value of scheduling variable f. In other words:

$$\begin{aligned}\left(\bar{A}(f) \quad \bar{B}(f)\right) &\in Co\left\{\left(\bar{A}_j \quad \bar{B}_j\right),j=1,\ldots,N\right\} \\ &:= \sum_{j=1}^{N}\alpha_j(f)\left(\bar{A}_j \quad \bar{B}_j\right),\end{aligned} \tag{23}$$

with $\alpha_j(f)\geq 0$ and $\sum_{j=1}^{N}\alpha_j(f)=1$.

Consequently, the LPV fault representation of system given by (22) can be expressed as follows:

$$\tilde{x}(k+1) = \sum_{j=1}^{N} \alpha_j(f)\left[\bar{A}_j \tilde{x}(k) + \bar{B}_j u(k)\right] - \bar{E} r(k), \qquad (24)$$

where $\alpha_j(f) = \alpha_j(f(k), k)$ and $f(k)$ is the value of f at the sample k, (see [17] for more details about LPV polytopic representation). Here $\bar{A}_j$ and $\bar{B}_j$ are constant matrices defined for j^{th} model, where each model is an admissible fault representation.

The polytopic system is scheduled through functions designed as follows: $\alpha_j(f), \forall j \in [1, \ldots, N]$ that lie in a convex set:

$$\Omega = \Big\{ \alpha_j(f) \in \mathbb{R}^N, \alpha(f) = [\alpha_1(f), \ldots, \alpha_N(f)]^T, \quad \alpha_j(f) \geq 0, \forall j, \sum_{j=1}^{N} \alpha_j(f) = 1 \Big\}. \qquad (25)$$

There are several ways to implement (24) depending on how $\alpha_j(f)$ functions are defined [20]. Here, the approach proposed in [21] is used:

$$\left(\bar{A}(f)\ \bar{B}(f)\right) = \sum_{j=1}^{N} \sum_{i_1=1}^{2} \cdots \sum_{i_{n_f}=1}^{2} \prod_{m=1}^{n_f} \mu_{m,i_m}(f_m) \left(\bar{A}_j\ \bar{B}_j\right) \qquad (26)$$

with $\mu_{m,1} = \frac{\left(f_m - \underline{f_m^j}\right)}{\left(\overline{f_m^j} - \underline{f_m^j}\right)}$ and $\mu_{m,2} = 1 - \mu_{m,1}$ where $\overline{f_m^j}$ and $\underline{f_m^j}$ represents the upper and lower bounds of f_m, respectively, and n_f is the number of scheduling variables (fault modes).

The polytopic formulation in (24) assumes that the effect of faults is included in the model through scheduling parameters $f(k) = [f_1(k), \ldots, f_{n_f}(k)]$ that evolve within known bounds: $\underline{f_j} \leq f_j(k) \leq \overline{f_j}, j = 1, ..., n_f$. From a practical point of view, these bounds can be prespecified to define the subset of the possible faults that must be tolerated by the FTC system.

Remark 2. Note that it is possible that the control performances specified through $\mathscr{M}$ can not be satisfied for all the range of faults defined by $[\underline{f_j}, \overline{f_j}]$. In this case, either the performances (less restrictive $\mathscr{M}$) or the range of tolerable faults should be reduced.

B. Motivation

The goal of the AMM FTC approach proposed in this paper is to maintain acceptable control performances under the presence of the pre-established set of faults. In case that the proposed approach is combined with an active strategy, once a fault has appeared, its magnitude will be estimated by the FDI module and the controller will be adapted accordingly (accommodation), trying to maintain acceptable performance. This leads to the control structure shown in Figure 2 that can be viewed as equivalent to a gain-scheduling control structure where the fault f is the scheduling variable and the FDI module is the parameter estimation algorithm. This suggests that gain-scheduling LPV theory can be used for the design of active AMM FTC considering that the faulty system behavior by the LPV fault representation introduced in (22).

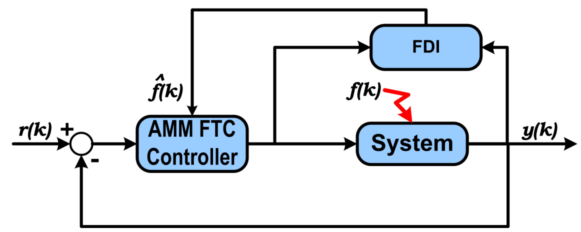

Fig. 2. Active AMM FTC for LPV fault representation where $\hat{f}(k)$ represents the fault magnitude estimation provided by FDI module.

If the state feedback control law (14) is used to control the system modelled using the LPV fault representation (22), then the FTC law is given by:

$$\begin{aligned} u(k) &= -[K_I(f)\ \ K(f)] \begin{bmatrix} x_I(k) \\ x(k) \end{bmatrix} + \bar{G}(f) r(k) \\ &= -\bar{K}(f)\tilde{x}(k) + \bar{G}(f) r(k) \end{aligned} \qquad (27)$$

And, consequently, the closed-loop behavior can be obtained as follows:

$$M = \bar{A}(f) - \bar{B}(f)\bar{K}(f) = \sum_{j=1}^{N} \alpha_j(f)\left[\bar{A}_j - \bar{B}_j \bar{K}_j\right] \qquad (28)$$

Then, the following definition can be introduced:

Definition 5. Admissibility using LPV fault representation: *The triple* $(\bar{A}(f), \bar{B}(f), \bar{K}(f))$ *is admissible if and only if it satisfies the condition* (4) *with* M *defined as* (28) *and is given by*

$$\mathscr{M}_a = \left\{ \left(\bar{A}(f), B(\bar{f}), \bar{K}(f)\right), f \in F : \Phi_{\mathscr{M}}\left(\bar{A}(f), \bar{B}(f), \bar{K}(f)\right) \leq 0 \right\} \qquad (29)$$

where $\Phi_{\mathscr{M}}\left(\bar{A}(f), \bar{B}(f), \bar{K}(f)\right)$ *is the set of constraints that guarantee that condition* (4) *is satisfied.*

The set $\mathscr{M}_a$ contains the set of admissible values $(\bar{A}(f), \bar{B}(f), \bar{K}(f))$ such that $(\bar{A}(f) - \bar{B}(f)\bar{K}(f)) \in \mathscr{M}$ can be achieved with the control law (27) and the LPV fault representation (22).

The constraints $\Phi_{\mathscr{M}}(\bar{A}(f), \bar{B}(f), \bar{K}(f))$ can be defined via LMIs leading to convex constraints. These LMIs can be designed for performance, stabilization and pole placement [12]. In this paper, the admissibility constraints can be defined using LMIs for the pole placement in LMI regions [11].

In case that the FDI module is not available, a single (robust) controller $\bar{K}$ is designed to maintain acceptable performance for the whole set of admissible faults should be used (passive FTC approach). Thus, Definition 5 should be modified considering that a single controller $\bar{K}$ is used instead of a gain-scheduling controller $K(\bar{f})$ used in the active AMM FTC approach.

C. LMI Pole Placement

As discussed above, the LMI pole placement algorithm proposed in [11] will be used in conjunction with the LPV fault representation of the system (22) to design a FTC such that guarantee that the closed-loop poles are inside a pre-established LMI region even in case of fault.

The main idea of the LMI pole placement algorithm is to design the state feedback gain K using LMIs such that the closed-loop poles of $(A-BK)$ lie in a suitable sub-region of the unit circle. According to [11], a subset $\mathscr{D}$ of the complex plane is called an LMI region if there exists a symmetric matrix $\alpha=[\alpha_{kl}]\in R^{m\times m}$ and a matrix $\beta=[\beta_{kl}]\in R^{m\times m}$ such that

$$\mathscr{D}=\{z\in C: f_{\mathscr{D}}(z)<0\} \tag{30}$$

where the characteristic function $f_{\mathscr{D}}(z)$ is given by $f_{\mathscr{D}}(z)=[\alpha_{kl}+\beta_{kl}z+\beta_{kl}\bar{z}]_{l\leq k,\, l\leq m}$ ($f_D(z)$ is valued in the space of $m\times m$ Hermitian matrices and "<0" stands for negative defined). Pole location in a given LMI region can be characterized in terms of the $m\times m$ block matrix as (See [11]):

$$\begin{aligned} M_{\mathscr{D}}(A,X) \quad := \quad & \alpha\otimes X+\beta\otimes(AX)+\beta^T\otimes(AB)^T \\ & [\alpha_{kl}+\beta_{kl}z+\beta_{kl}\bar{z}]_{l\leq k,\, l\leq m} \end{aligned} \tag{31}$$

where the matrix A is $\mathscr{D}$-stable if and only if there exists a symmetric matrix X such that:

$$M_{\mathscr{D}}(A,X)<0 \qquad X>0 \tag{32}$$

Note that $M_{\mathscr{D}}(A,X)$ in (31) and $f_{\mathscr{D}}(z)$ are related by the substitution $(X,\ AX,\ XA^T)\leftrightarrow(1,z,\bar{z})$. Then, the disk of radius r and center $(-q,0)$ is an LMI region with characteristic function

$$f_{\mathscr{D}}(z)=\begin{bmatrix} -r & q+z \\ q+\bar{z} & -r \end{bmatrix} \tag{33}$$

Thus, Eq. (32) leads to the following LMI:

$$\begin{bmatrix} -rX & qX+AX \\ qX+XA^T & -rX \end{bmatrix}<0 \tag{34}$$

1) Active AMM FTC Design using LMI Region: As already discussed, in case that a FDI module is available, an active AMM FTC control law can be obtained with the pole placement in a LMI region combined with the LPV fault representation.

Admissibility condition for active AMM FTC: The set of admissible behaviors $\mathscr{M}_a$ can be proposed as:

$$\mathscr{M}_a=\left\{\left(\bar{A}(f),\bar{B}(f),\bar{K}(f)\right):\lambda\left(\bar{A}(f)-\bar{B}(f)\bar{K}(f)\right)\in\mathscr{D}_\alpha\right\} \tag{35}$$

where $\mathscr{D}_\alpha$ is a desired region LMI included in the unit circle with an affix $(-q,0)$ and a radius r such that $(q+r)<1$ is fixed. These two scalars q and r are used to determine a specific region included in the unit circle. According to [13] and [11], the LMI that should be solved to guarantee that the poles will be in this LMI region is the following:

$$\begin{bmatrix} -rX_j & qX_j+(M_jX_j)^T \\ (q+M_jX_j) & -rX_j \end{bmatrix}<0, \tag{36}$$

where $x(k+1)=M_jx(k)$ for each j^{th} model obtained from the LPV fault representation of the system given in (25). More precisely, each model M_j is defined as:

$$M_j=\bar{A}_j^f-\bar{B}_j^f\bar{K}_j \tag{37}$$

where $\bar{A}_j^f$ and $\bar{B}_j^f$ define the set of admissible parametric faults and corresponds the vertices of LPV fault representation presented in Section III-A. Thus, the inequalities (37) can be rewritten as follows:

$$\begin{bmatrix} -rX_j & qX_j+X_j^T((\bar{A}_j^f)^T-\bar{K}_j^T(\bar{B}_j^f)^T) \\ (q+\bar{A}_j^f-\bar{B}_j^f\bar{K}_j)X_j & -rX_j \end{bmatrix}<0, \tag{38}$$

for all $j\in[1,\ldots,N]$. Thus, by substituting $W_j=\bar{K}_jX_j$ it can be shown that:

$$\begin{bmatrix} -rX_j & qX_j+X_j^T(\bar{A}_j^f)^T-W_j^T(\bar{B}_j^f)^T \\ (q+\bar{A}^f)X_j-\bar{B}_j^fW_j & -rX_j \end{bmatrix}<0, \tag{39}$$

The design procedure boils down solving the LMI (39), and then determining the set of gains $\bar{K}_j=W_jX_j^{-1}$. The design of $\bar{G}_j$ can be solved as:

$$\bar{G}_j=K_jN_x \tag{40}$$

where N_x is calculated by satisfying the condition (17) and K_j is obtained from the augmented model (13) as: $\bar{K}_j=[K_{I,j}\ K_j]$.

Finally, the active AMM FTC control law of the system (24) according to (27) is given by:

$$u(k) \quad = \quad \sum_{j=1}^{N}\alpha_j(f)\left[\bar{G}_jr(k)-\bar{K}_j\tilde{x}(k)\right] \tag{41}$$

Remark 3. Note that the solutions of the LMIs (39) is done off-line and the evaluation of (41) just requires the computation of $\alpha_j(f)$ according to (26). Due to the simplicity of this computation, the real-time implementation of the controller reconfiguration is possible.

Remark 4. [22] proposes a set admissible behaviors $\mathscr{M}$ using LMI regional pole placement. This method locates the poles in particular convex regions called $\mathscr{D}$-regions. The fault accommodation is formulated in terms of several LMI problems. Using this approach, the *admissibility condition* could also be defined using the set admissible behaviors $\mathscr{M}$ proposed by [22] and the LMI regional pole placement approach.

2) Passive AMM FTC Design using LMI Region: On the other hand, in case that the FDI module is not implemented, a passive AMM FTC control law can analogously be obtained using the pole placement in a LMI region.

Admissibility condition for passive AMM FTC: The set of admissible behaviors $\mathscr{M}_a$ is defined as follows:

$$\mathscr{M}_a=\left\{\left(\bar{A}_f,\bar{B}_f,\bar{K}\right):\lambda\left(\bar{A}_f-\bar{B}_f\bar{K}\right)\in\mathscr{D}_\alpha\right\} \tag{42}$$

Analogously to the active FTC, a passive FTC control law can be solved with the pole placement in LMI region of the closed-loop system for all admissible faults of the models $j\in[1,\ldots,N]$ defining M_j as:

$$M_j=\bar{A}_j^f-\bar{B}_j^f\bar{K}. \tag{43}$$

where $M_j\in\mathscr{M}$ for $j=1,\ldots,N$

Proceeding in the same way than in the case (37), but substituting $W=\bar{K}X$ it is possible to obtain:

$$\begin{bmatrix} -rX & qX+X^T(\bar{A}_j^f)^T-W^T(\bar{B}_j^f)^T \\ (q+\bar{A}_j^f)X-\bar{B}_j^fW & -rX \end{bmatrix}<0, \tag{44}$$

The design procedure boils down solving the set of N-LMIs (44) by determining $\bar{K}=WX^{-1}$ and $\bar{G}=KNx$. Finally, consider the gain $\bar{K}$ to calculate the control law according to (14) that can be controlled the system (13).

IV. Characterization of Recoverability using LPV Theory

The proposed active AMM FTC assumes that the pair $(\bar{A},\bar{B})$ depends on the fault f as $(\bar{A}(f),\bar{B}(f))$. This allows to specify the set of fault $f \in F$ that must be tolerated by the FTC controller.

For the pair $(\bar{A}(f),\bar{B}(f))$ is necessary to find an appropriate $\bar{K}(f)$ that is reconfigured according to the fault type and magnitude such that the triple $(\bar{A}(f),\bar{B}(f),\bar{K}(f))$ is admissible. From the Definition 5 of admissibility using LPV representation, the following definitions can be introduced:

Definition 6. Recoverability using LPV fault representation: *The system* (24) *that includes the faults as scheduling variables is recoverable by the whole set of faults $f \in F$ if and only if the set:*

$$\mathscr{K}\left(\bar{A}(f),\bar{B}(f)\right) = \left\{K(f) : \Phi_{\mathscr{M}}\left(\bar{A}(f),\bar{B}(f),\bar{K}(f)\right) \leq 0\right\} \qquad (45)$$

is not empty. That means the reconfigurable gains exist $\bar{K}(f)$ such that $(\bar{A}(f)-\bar{B}(f)\bar{K}(f)) \in \mathscr{M}$ for all $f \in F$.

Definition 7. Recoverable faults using LPV fault representation: *The set $\mathscr{R}(\mathscr{M}_a)$ of all* recoverable faults *in the active AMM FTC is:*

$$\mathscr{R}(\mathscr{M}_a) = \left\{\left(\bar{A}(f),\bar{B}(f)\right), f \in F : \mathscr{K}\left(\bar{A}(f),\bar{B}(f)\right) \neq \varnothing\right\} \qquad (46)$$

Definitions 6 and 7 apply to active AMM FTC approach. Analogously, it can be easyly adapted for the passive AMM FTC approach replacing the reconfigurable gain $\bar{K}(f)$ by a single gain K. This gain $\bar{K}$ should satisfy $(\bar{A}(f)-\bar{B}(f)\bar{K}) \in \mathscr{M}$ for all $f \in F$.

A. Recoverability evaluation using LMI regions

Consider the set of faults that must be tolerated $f \in F$ and the *admissibility condition* (35) defined in Section III-B. Then, the recoverability evaluation by means of LMI regions involves finding a set of $\bar{K}(f)$:

$$\mathscr{K}_a\left(\bar{A}(f),\bar{B}(f)\right) = \left\{\bar{K}(f) : \lambda\left(\bar{A}(f)-\bar{B}(f)\bar{K}(f)\right) \in \mathscr{D}_\alpha\right\} \qquad (47)$$

where the system $\left(\bar{A}(f),\bar{B}(f)\right)$ is recoverable if and only if $\mathscr{K}_a\left(\bar{A}(f),\bar{B}(f)\right) \neq \varnothing$ according to Definition 6.

The solution to problem (47) provides the gains $\bar{K}(f)$, where the eigenvalues $\lambda\left(\bar{A}(f)-\bar{B}(f)\bar{K}(f)\right)$ are in stability $\mathscr{D}_\alpha$ region of the unit circle.

Proceeding in the same way as in the case (47), the recoverability evaluation using LMI regions for passive AMM FTC relies on finding a set of $\bar{K}$:

$$\mathscr{K}_p\left(\bar{A}(f),\bar{B}(f)\right) = \left\{\bar{K} : \lambda\left(\bar{A}(f)-\bar{B}(f)\bar{K}\right) \in \mathscr{D}_\alpha\right\} \qquad (48)$$

where the system $(\bar{A}(f),\bar{B}(f))$ is recoverable if and only if $\mathscr{K}_p(\bar{A}(f),\bar{B}(f)) \neq \varnothing$.

B. Recoverability design using LMI regions and optimization

The gains $K(f)$ in $\mathscr{K}_a$ are admissible solutions. But, it is still necessary to establish some design criteria to select a particular gain in order to accommodate the control loop.

Consider the set of faults that must be tolerated $f \in F$, the optimal desired behavior M^* and the *admissibility condition* (35). Then, the recoverability design using LMI regions consists on finding the optimal gain $\bar{K}(f)$ as follows:

$$\bar{K}^{Opt}(f) = \arg\min_{K(f)} J(K(f)) \qquad (49)$$

such that

$$\lambda\left(\bar{A}(f)-\bar{B}(f)\bar{K}(f)\right) \in \mathscr{D}_\alpha$$

This optimization problem with LMIs constraints can be solved with YALMIP [23]. The objective $J(K(f))$ and specifications of the optimization problem that can be included are: H_2/H_∞ performance, time-domain constraints (peak amplitude, overshot, settling time), regulation, among others [24].

Analogously, a passive AMM FTC can be designed in a similar way than the active case but considering the *admissibility condition* (42). Thus, the recoverability design implies findidng the optimal gain $\bar{K}$ by solving the following optimization problem:

$$\bar{K}^{Opt} = \arg\min_{\bar{K}} J(K(f)) \qquad (50)$$

such that

$$\lambda\left(\bar{A}(f)-\bar{B}(f)\bar{K}\right) \in \mathscr{D}_\alpha$$

C. Set of recoverable faults using LMI regions

It can be interesting to discover the limits of applicability of the *passive AMM Fault Control Tolerance* for a given system $(\bar{A}(f),\bar{B}(f))$ under the control law $\bar{K}$. This will allow to establish a criteria to know for which fault size accommodation is necessary. Considering the Definition 7 in the passive AMM FTC context, the set of faults that can be tolerated $f \in F$ can be determined as:

$$\max_{f \in F} \|(\Delta A, \Delta B)\|_\infty \qquad (51)$$

such that

$$\lambda\left(\bar{A}(f)-\bar{B}(f)\bar{K}\right) \in \mathscr{D}_\alpha$$

for a given $\bar{K}$, where $\Delta A = \bar{A}(f)-\bar{A}$ and $\Delta\bar{B} = \bar{B}(f)-\bar{B}$. The solution of this problem takes into account the *admissibility condition* (42).

In the case, that any fault can not be recoverable using the passive AMM FTC approach, an active AMM FTC controller should be used instead. From Definition 7, the set of faults that can be tolerated $f \in F$ by an active controller can be determined by replacing the constraints in (51) by $\lambda\left(\bar{A}(f)-\bar{B}(f)\bar{K}(f)\right) \in \mathscr{D}_\alpha$.

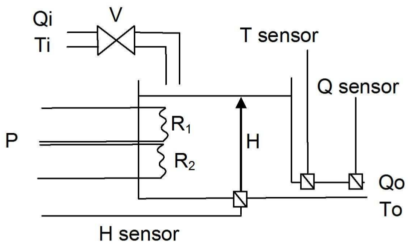

Fig. 3. Thermal Hydraulic System

V. Design example

A. Process description

The AMM FTC approach based on the LPV fault representation has been applied to a thermal hydraulic system (see Figure 3). The goal of the process is to assure a constant water flow rate Q_0 with a given temperature T_0.

The process is composed of a tank equipped with two heating resistors R_1 and R_2. The inputs are the water flow rate Q_i, the water temperature T_i and the heater electric power P. The outputs are the water flow rate Q_0 and the temperature T_0 which is regulated around an operating point. The temperature of the water T_i is assumed to be constant.

The system can be represented by the following equations:

$$\begin{aligned} S\frac{dh(t)}{dt} &= q_i(t) - q(t) \\ \frac{dT(t)}{dt} &= \frac{P}{\mu CSh(t)} - \frac{(T(t)-T_i)Q_i}{Sh(t)} \end{aligned} \quad (52)$$

$q(t) = \alpha\sqrt{h(t)}$, $T_i = 20^oC$, $S = 1$, $\mu C = 2\cdot 10^6$ and $\alpha = \frac{20(10^{-3})}{60\sqrt{0.6}}$.

The system (52) is linearized around the operating point given by $q_{in}^{op} = q_{out}^{op} = \alpha\sqrt{h_{op}}$, $P_{op} = \mu C(q_{in}^{op})(T_{op}-T_i)$, $T_{op} = 50^oC$, $h_{op}=0.6$. In this example, the level of the water h is used instead of the water flow rate Q_0 to obtain the linear model. With the previous conditions, the linear system in fault-free case can be specified by:

$$\begin{aligned} \dot{x}(t) &= A_c x(t) + B_c u(t) \\ y(t) &= Cx(t) \end{aligned} \quad (53)$$

with:

$$A_c = \begin{bmatrix} \frac{-q_{out}^{op}}{2Sh_{op}} & 0 \\ 0 & \frac{-q_{out}^{op}}{Sh_{op}} \end{bmatrix}, \quad B_c = \begin{bmatrix} \frac{1}{S} & 0 \\ \frac{T_{op}-T_i}{Sh_{op}} & \frac{1}{\mu CSh_{op}} \end{bmatrix}, \quad C = I$$

Considering a sampling time equal to $T_s = 360s$, the system (52) can be expressed in discrete time as:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \quad (54)$$

where $A = T_sA_c + I$ and $B = T_sB_c$ using the Euler approximation.

The model is augmented including an integrator as in (13) for set-point tracking purposes:

$$\tilde{x}(k+1) = \bar{A}\tilde{x}(k) + \bar{B}u(k) - \bar{E}r(k), \quad (55)$$

where:

$$\bar{A} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a_1 & 0 \\ 0 & 0 & 0 & a_2 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_1 & 0 \\ b_2 & b_3 \end{bmatrix},$$

$$\bar{E} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad r(k) = \begin{bmatrix} T_0 \\ h \end{bmatrix},$$

with $a_1=0.9$, $a_2=0.8$, $b_1=360$, $b_2=-18000$ and $b_3=0.0003$.

The considered parametric faults are expressed as changes in the system dynamics (54):

$$A = \begin{bmatrix} a_1 \pm f_{a1} & 0 \\ 0 & a_2 \pm f_{a2} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \pm f_{b1} & 0 \\ b_2 \pm f_{b2} & b_3 \pm f_{b3} \end{bmatrix}.$$

In this application example, only a fault at a time has been considered just to illustrate the effectiveness of the proposed AMM FTC strategy. However, it can also be applied to multiple faults.

The desired nominal closed-loop poles of the controller are:

$$\lambda_i^* = \{0.3\} \quad \forall i \in [1,\dots,4] \quad (56)$$

Finally, a closed-loop behavior is considered admissible if the eigenvalues of M lie in a disk around 0.3 with a radius of 0.3 that corresponds to a 30% degradation of the nominal closed-loop specifications:

$$\mathscr{M} = \{M : \lambda_i\left(\bar{A} - \bar{B}\bar{K}\right) \in \mathscr{D}_a(\lambda_i^*, 0.3)\} \quad \forall i \in [1,\dots,4] \quad (57)$$

In order to show the effectiveness of the proposed approach, a passive and active AMM FTC controllers will be compared with a nominal controller designed using standard pole placement tools.

B. Set of recoverable faults

To determine the set of recoverable faults for a given controller and a given the set of admissible behavier $\mathscr{M}$, the AMM FTC design procedure is applied by increasing iteratively the fault size. Then, the maximum fault size is reached when the AMM FTC design problem has no solution.

1) Nominal controller: The nominal controller is designed using standard pole place tools using the augmented model (55) and considering that the desired poles are (56). Then, the gain of the control law (14) is $\bar{K}^n = [K_I^n \; K^n]$:

$$K_I^n = \begin{bmatrix} 0.0014 & 0 \\ 81875 & 1713 \end{bmatrix}, \quad K^n = \begin{bmatrix} 0.0036 & 0 \\ 216965 & 4114 \end{bmatrix}.$$

The recoverable faults of the nominal controller are evaluated by the solution the optimization problem (51). To solve this problem, the admissible specifications (57) are considered. The size of the recoverable faults are shown in Table 1 for parametric faults in matrix A and in Table 2 for parametric faults in matrix B.

Finally, the obtained closed-loop pole migration under the considered set of faults is drawn in Figure 4 where the region of admissible eigenvalues assignment $\mathscr{D}_\alpha$ is also represented.

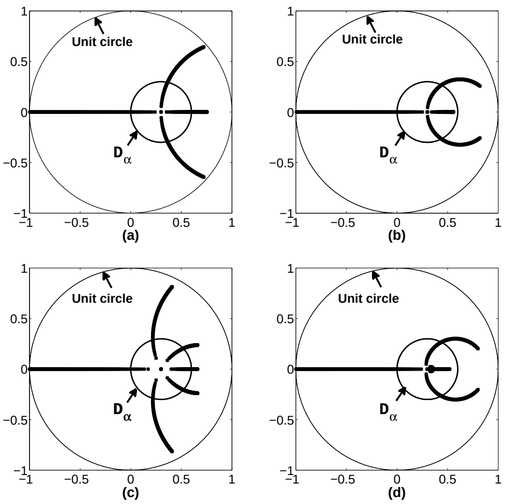

Fig. 4. Closed-loop pole migration for different fault sizes for nominal controller. The recoverable fault are shown in Table 1. (a) Fault f_{a1} or f_{a2} (b) Fault f_{b1} (c) Fault f_{b2} (d) Fault f_{b3}

2) Passive Controller: An passive FTC controller has been designed using the method presented in Section III-C using the pole placement in LMI regions (44) to guarantee that the closed-loop behaviour under the fault effect is in the region.

The recoverable faults set corresponding to this controller is evaluated by the solution of the optimization problem (51). The results are presented in Table 1 and 2. The closed-loop poles evolution varying the fault size for each parameter is shown in Figure 5.

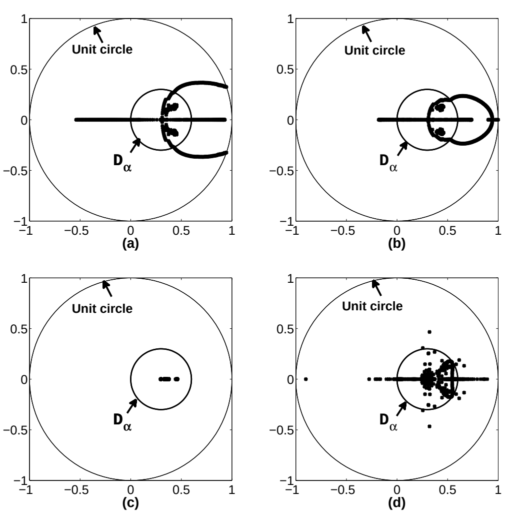

Fig. 5. Closed-loop pole migration for different faults for passive controller. The recoverable faults are shown in Table 1. (a) Fault f_{a1} or f_{a2} (b) Fault f_{b1} (c) Fault f_{b2} (d) Fault f_{b3}

3) Active Controller: Finally, an active FTC controller has been designed using the LPV fault representation (24) and the control law (41). The gain $\bar{K}(f)$ is obtained using the LMIs (39) and vertex matrices are the same as in the passive case. It is assumed that the FDI module is ideal providing a perfect instantaneous fault magnitude estimation.

As in the case of the nominal and passive controllers, the recoverable faults are calculated and presented in Table 1 and 2. The set of recoverable faults are obtained by solving the optimization problem (51) for the active case. In this table, *no limits* corresponds to the case the FTC system performance is admissible for any fault size. The pole migration for different fault sizes of f_{a1}, f_{a2} and f_{b2} are the same as those shown in Figure 5(c), while for the faults f_{b1} and f_{b3} is drawn in Figure 6.

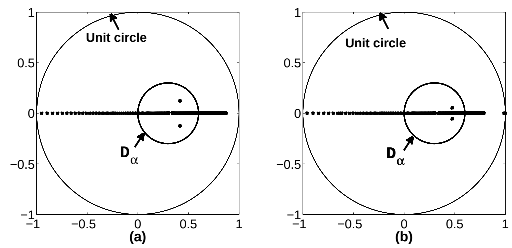

Fig. 6. Closed-loop pole migration for different faults for active controller. The recoverable faults are shown in Table 1. (a) Fault f_{b1} (b) Fault f_{b3}

4) Comparison of Controllers: Analyzing the results of Table 1 and 2 corresponding to the set of recoverable faults for the different controllers, the following conclusions can be extracted: The nominal controller is designed for the fault-free system. Therefore, this controller accepts less variation in the parameters of matrices A and B than the two others. On the other hand, the active controller allows parameters a_1, a_2 and b_2 vary without limits. While parameters b_1 and b_3 can vary in a larger interval than the nominal and passive controllers.

Remark 5. To select between the passive and active FTC controller, it is necessary to analyze the set of recoverable faults (Table 1). For example, if the FTC should tolerate faults that change the parameter a_1 up to 15%, according to Table 1, the passive FTC controller would be enough. But, if a 50% of change in the parameter a_1 was the fault tolerant requirement, the active FTC controller should be used instead.

TABLE I
RECOVERABLE FAULTS IN THE PARAMETERS OF MATRIX A.

Parameter	Nominal	Passive	Active
$a_1 = 0.9$	$a_1(1 \pm 0.1)$	$a_1(1 \pm 0.15)$	no limits
$a_2 = 0.8$	$a_2(1 \pm 0.11)$	$a_2(1 \pm 0.16)$	no limits

TABLE II
RECOVERABLE FAULTS IN THE PARAMETERS OF MATRIX B.

Parameter	Nominal	Passive	Active
b_1=360	$b_1(1 \pm 0.11)$	$b_1(1 \pm 0.21)$	$b_1(1 \pm 0.33)$
b_2=-18000	$b_2(1 \pm 0.19)$	no limits	no limits
b_3=3e-4	$b_3(1 \pm 0.13)$	$b_3(1 \pm 0.43)$	$b_3(1 \pm 0.67)$

C. Fault scenarios

To illustrate the effectiveness of the proposed AMM FTC approach, two fault scenarios are presented. The fault scenario 1 shows the performance of the controllers when the fault affects a parameter of the matrix A. The fault scenario 2 presents the fault effect in matrix B. In these scenarios, the temperature response does not show significant changes. In the following, the results corresponding only to level response will be analyzed. According with these results, the active AMM FTC controller tolerates a set of admissible faults bigger than the other controllers.

1) Fault scenario 1: A fault in the parameter a_1 of matrix A has been simulated. Table 3 shows the set of faults to be tolerated that has been considered in the design of the controllers and the particular fault magnitude used in the plots of Figure 7.

TABLE III
SET OF FAULTS TO BE TOLERATED USING IN THE DESIGN OF EACH CONTROLLER AND CONSIDERED FAULT SCENARIOS

a_1=0.9	Nom.	Passive FTC	Active FTC	Fault
Figure	a_1	$a_1 \pm f_{a1}$	$a_1 \pm f_{a1}$	a_1^f
7(a)	0.9	[0.774, 1.026]	[-0.45, 2.25]	0.98
7(b)	0.9	[0.774, 1.026]	[-0.45, 2.25]	1.53
7(c)	0.9	[0.774, 1.026]	[-0.45, 2.25]	1.8

The first scenario (a_1^f=0.98) is shown in Figure 7(a), the fault is admissible for all controllers and their temporal responses are similar. The Figure 7(b) shows the second scenario (a_1^f=1.53), where the fault affects the performance of nominal and passive controller since this fault is out of their set of recoverable faults. The third fault (a_1^f=1.8) is shown in Figure 7(b) where it can be seen that the nominal and passive controller can not stabilize the system under this fault. On the other hand, the active controller achieves the desired performance.

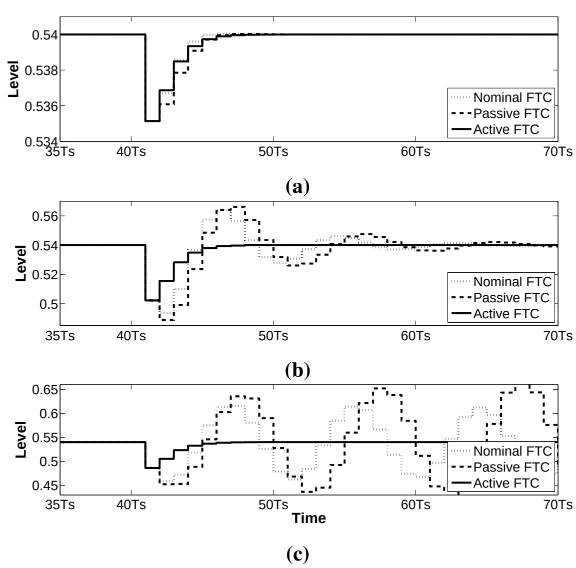

Fig. 7. Level (in m) of the water response with fault in $k = 40T_s$. The fault parameter a_1 of: (a) $a_f = 0.981$, (b) $a_f = 1.53$ and (c) $a_f = 1.8$

2) Fault scenario 2: In this scenario the faulty parameter is b_1. Table 4 presents the set of faults that should be tolerated by design and the parameter fault magnitude considered in each scenario.

TABLE IV
SET OF FAULTS TO BE TOLERATED USING IN THE DESIGN OF EACH CONTROLLER AND CONSIDERED FAULT SCENARIOS

b_1=360	Nom.	Passive FTC	Active FTC	Fault
Figure	b_1	$b_1 \pm f_{b1}$	$b_1 \pm f_{b1}$	b_f
8(a)	360	[284.4, 435.6]	[180, 540]	324
8(b)	360	[284.4, 435.6]	[180, 540]	526
8(c)	360	[284.4, 435.6]	[180, 540]	702

The faulty scenario corresponding to b_1=324 is shown in Figure 8(a). The fault is recoverable for all controllers and their temporal responses are similar with a slower response in passive controller case. The Figure 8(b) shows the faulty scenario corresponding to b_1=576, where the nominal and passive controllers are out of the interval of faults to be toleranted. However, the passive controller provides still an acceptable response while the performance of nominal controller is unacceptable. Finally, the scenario corresponding to b_1=612 is shown in Figure 8(c) where the nominal and passive controller can not stabilize this faulty system. The active controller provides in all scenarios the desired performance although in the third case was not designed for that fault magnitude.

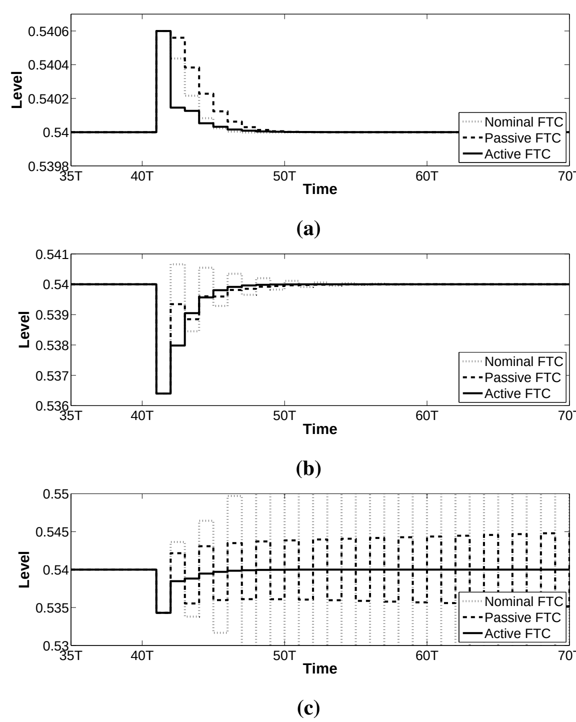

Fig. 8. Level (in m) of the water response with fault in $k = 40T_s$. The fault parameter b_1 of: (a) $b_f = 324$, (b) $b_f = 576$ and (c) $b_f = 702$

VI. CONCLUSIONS

In this paper, a new approach to design an AMM FTC has been proposed based on LPV fault representation. The *active AMM FTC* proposed uses a LPV controller where the scheduling variables are the parametric faults. Under these assumptions, the advantage of the approach is that allow the redesign of the controllers online by using a set of pre-established admissible faults. When the fault is in this recoverable fault interval, the system can be recovered with the performance desired.

If the fault estimation is not available, a *passive AMM FTC* approach can be used following the same ideas than the active version. Passive approach determines a single controller that is able to cope with set of considered admissible faults. The drawback is that the size of the admissible faults is smaller compared to the active case.

The set of admissible behaviors are designed in terms of the pole placement assignment using LMI regions. This procedure allows a more versatile and straightforward representation of the admissible closed-loop behavior of the faulty plant that the FTC should guarantee.

The recoverability evaluation of proposed FTC approach has also been characterized by means of pole placement LMI regions. First, a set of gains $\mathscr{K}$ that guarantee the recoverability of the whole set of fault $f \in F$ is obtained. Second, a optimal gain $\bar{K}(f)$ for the active approach or $\bar{K}$ for the passive approach can be calculated. This solution is obtained by solving an optimization problem that can include as objective (specifications): H_2/H_{inf} performance, time domain constraints (peak amplitude, overshoot, settling time), regulation, among others.

To select between the active or passive AMM FTC approach, the set of recoverable faults that can be tolerated by each approach is proposed. If the recoverability of the passive AMM FTC is evaluated and the set of recoverable faults is empty, an active AMM FTC should be implemented instead. In this case, the parametric fault should be estimated by using an FDI module. If even with an active AMM FTC the control performances specified through $\mathscr{M}$ could not be satisfied for all set of fault $f \in F$, then either the admissible behavior $\mathscr{M}$ or the set of faults to be tolerated should be reduced.

As a future work, this approach will be extended to non-linear systems that can be approximated by an LPV model.

VII. Acknowledgments

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References

[1] R. J. Patton, "Fault-Tolerant Control Systems: The 1997 situation," *Proceedings of the IFAC Symposium: SAFEPROCESS'97, Hull, UK.*, vol. 2, pp. 1033–1055, 1997.

[2] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki, *Diagnosis and Fault-Tolerant Control*. Springer-Verlag Berlin Heidelberg, 2003.

[3] J. Chen, R. J. Patton, and Z. Chen, "An LMI Approach to Fault-Tolerant Control of Uncertain Systems," in *IEEE Conference on Decision and Control*, vol. 1, 1998, pp. 175–180.

[4] Y. Zhang, L. Guo, H. Yu, and K. Zhao, "Fault Tolerant Control based on Stochastic Distributions via MLP Neural Networks," *Neurocomputing*, vol. 70, pp. 867–874, 2007.

[5] Y. Zhang and J. Jiang, "Bibliographical review on reconfigurable fault-tolerant control systems," *Annual Reviews in Control*, vol. 32, no. 2, pp. 229–252, December 2008.

[6] H. Noura, D. Theilliol, J. C. Ponsart, and A. Chamseddine, *Fault-tolerant control systems: Design and practical applications*. Springer London, 2009.

[7] A. J. Ostroff, "Techniques for Accommodating Control Effector Failures on a mildly Statically Unstable Airplane." *American Control Conference*, pp. 903–906, 1985.

[8] M. Staroswiecki, "Fault Tolerant Control: The Pseudo-Inverse Method Revisited," *IFAC World Congress, Prague, Czech Republic*, 2005.

[9] ——, "Fault Tolerant Control using an Admissible Model Matching Approach," *44th IEEE Conference on Decision and Control - European Control Conference*, 2005.

[10] ——, "Robust Fault Tolerant Linear Quadratic Control based on Admissible Model Matching," *45th IEEE Conference on Decision and Control*, pp. 3506–3511, 2006.

[11] M. Chilali and P. Gahinet, "H_∞ Design with Pole Placement Constraints: An LMI Approach," *IEEE Transactions on Automatic Control*, vol. 41, no. 3, pp. 358–367, 1996.

[12] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. SIAM Studies in Applied Mathematics, 1994.

[13] P. Apkarian, P. Gahinet, and G. Becker, "Self-scheduled H_∞ Control of Linear Parameter-Varying Systems: A Design Example," *Automatica*, vol. 31, no. 9, pp. 1251 – 1261, 1995.

[14] J. Bokor, Z. Szabo, and G. Stikkel, "Failure Detection for Quasi LPV Systems," *Proceedings of the 41st IEEE Conference on Decision and Control*, vol. 3, pp. 3318–3323, 2002.

[15] J. Bokor, "Fault Detection and Isolation in Nonlinear Systems," *7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes (Plenary Session) 2009, Barcelona, Spain*, 2009.

[16] N. E. Wu and J. Jianhong, "Parametric Modeling and Fault Tolerant Control," *Digital Avionics Systems Conferences*, vol. 2, pp. 6F3/1–6F3/8, 2000.

[17] M. Rodrigues, D. Theilliol, S. Aberkane, and D. Sauter, "Fault Tolerant Control Design for Polytopic LPV Systems," *International Journal of applied Mathematics and Computer Science*, vol. 17, no. 1, pp. 27–37, 2007.

[18] F. Guenab, D. Theilliol, P. Weber, Y. Zhang, and D. Sauter, "Fault Tolerant Control System Design: A Reconfiguration Strategy based on Reliability Analysis under Dynamic Behaviour Constraints," *6th IFAC Symposium on Fault Detection Supervision and Safety for Technical Processes SAFEPROCESS*, 2006.

[19] G. F. Franklin, J. D. Powell, and M. L. Workman, *Digital Control of Dynamic Systems*, 3rd ed. Addison Wesley Longman, 1997.

[20] R. Murray-Smith and T. A. Johansen, *Multiple Model Approaches to Modelling and Control*. Taylor and Francis, 1997.

[21] P. Baranyi, D. Tikk, Y. Yam, and R. J. Patton, "From Differential Equations to PDC Controller Design via Numerical Transformation," *Computers in Industry*, vol. 51, no. 3, pp. 281–297, 2003.

[22] S. Tornil-Sin, J. C. Ponsart, D. Theilliol, and V. Puig, "A New Formulation for Admissible Model Matching in Fault-Tolerant Control," *23rd IAR Workshop on Advanced Control and Diagnosis*, 2008.

[23] J. Löfberg, "YALMIP : A Toolbox for Modeling and Optimization in MATLAB," in *Proceedings of the CACSD Conference*, 2004.

[24] C. W. Scherer, P. Gahinet, and M. Chilali, "Multi-objective output feedback control via LMI optimization," *IEEE Transactions on Automatic Control*, vol. 42, pp. 896–911, 1997.